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FABRICATION OF STRUCTURAL SPECIMENS FOR THE INVESTIGATION OF PEAK PRESSURE AND MULTI-MODE EFFECTS OF SONIC FATIGUE

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PACIFIC APPLIED RESEARCH

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FABRICATION OF STRUCTURAL SPECIMENS FOR THE INVESTIGATION OF PEAK PRESSURE AND MULTI-MODE EFFECTS OF SONIC FATIGUE
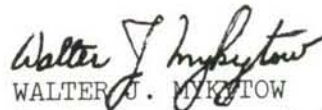
PACIFIC APPLIED RESEARCH

ABSTRACT

This report describes the analysis, design and fabrication of structural specimens to be used in an investigation of peak pressure and multi-mode effects of sonic fatigue. These specimens are to be fatigue tested to failure in the RTD Sonic Fatigue Facility by RTD personnel.

Details of the analysis and design are presented for three-bay panels and frame stringer panels. The three-bay panels were designed to fatigue in 10^5 cycles when exposed to an SPL of 162 db. A linear structural response approach is used, and the possibility of nonlinearities is examined.

The frame stringer panels were designed to withstand various combinations of internal bursting pressures and external aerodynamic loads.

Procedures used in fabricating the above specimens, and specimens designed by R&D personnel are also described.

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LIST OF ILLUSTRATIONS

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LIST OF SYMBOLS

A	area, in^2
A_m	modal amplitude
D	$Eh^3/12\,(1-\nu^2)$
E	Young's modulus, psi
F_s	modal force (static)
I	bending moment of inertia
I_a	square of the acoustic pressure
K	modal stiffness
M	modal mass
MF	modal force
M_x	bending moment
N	number of stringers
P	force, lb
P_s	static pressure
PSD(f)	power spectral density of the acoustic pressure (p^2/cps)
Q	quality factor
R	radius, inches
S_{rms}	root mean square stress
S_s	static stress
S_x	stress in the X direction
T	kinetic energy
V	potential energy
V_i	shear at point i, lb.
W	distributed load
Z	section modulus, I/C

a	length of a panel
b	width of a panel
f	frequency (cps)
h	thickness
i,j	subscripts
k	spring constant, lb/in
l_x	length along the X coordinate, inches
m	mass/unit length
p	pressure, psi
q	distributed load, lb/in
s	distance measured on deflected panel surface
t	thickness, inches
w	deflection
x, y, z	coordinates
Δf	test range (cps)
ω	frequency, radians/sec
ν	Poisson's ratio
Ω	frequency, radians/sec
Ψ	mode shape
ρ	mass/unit volume
ρ'	mass/unit area
ξ	modal displacement
σ	stress, psi

1. INTRODUCTION

Sonic fatigue is a major reason for failure in modern, high speed vehicles. For this reason a good engineering understanding and an accurate method of fatigue prediction, is necessary in the design and development of new systems. The Sonic Fatigue Facility at Wright-Patterson Air Force Base has been designed as an experimental tool to advance the state of the art in this area. This report describes the fabrication and design of test specimens constructed for use in this facility.

The test specimens are divided into two general categories. The first category consists of cantilevered beams and two bay panels. These specimens were completely designed by Air Force personnel. The construction of these specimens is discussed in Sections 4.1 and 4.2. The second category consists of three bay panels and frame stringer panels. In this category, the final design was accomplished by PAR personnel based on Air Force furnished design criteria.

The three bay panels were designed using linear theory and a fatigue life criterion of 10^5 cycles. The effects of non-linearities were examined. The analytical work resulting in the final design is presented in Section 2 and a description of the fabrication is given in Section 4.3.

The frame stringer panels are designed to withstand various combinations of internal and external static pressures. The design work is presented in Section 3 and the fabrication is described in Section 4.4.

2. DESIGN OF THE THREE BAY PANEL

The three bay panel (Fig. 1) is designed using linear theory and a fatigue life criteria of 10^5 cycles. The panel is to be exposed to normal incident sound with pressure levels of 162 db, the energy being equally distributed over a band width three times the frequency separation of the half-power points of the modal response of interest.

In addition, the effect of membrane stress on the S-N curve for sinusoidal loading is derived. Random fatigue curves are then obtained using Miles' theory which assumes a Rayleigh peak distribution. The effects of this assumption are discussed.

2.1 DYNAMIC RESPONSE AND STRESS ANALYSIS

The mode shapes to be considered in this analysis are those shown in Figure 2a and 2b.

The area where fatigue is desired is near the stiffeners. In addition the region of highest stress is in the area of greatest bending. Therefore, the modes of interest are those shown in Figure 2a and 2c. Figure 2c will occur rather then 2b when the Z section is relatively stiff in rotation.

Since the plate is to be excited by a plane wave, the modal force (MF) can be expressed as:

$$MF = \int_{area} p \, \Psi(z) \, dz$$

where
$$p = \text{pressure}$$

$$\Psi(z) = \text{mode shape}$$

By examination it can be seen that the modal force for the mode shape shown in Figure 2a is approximately three times that of Figure 2c. Based on
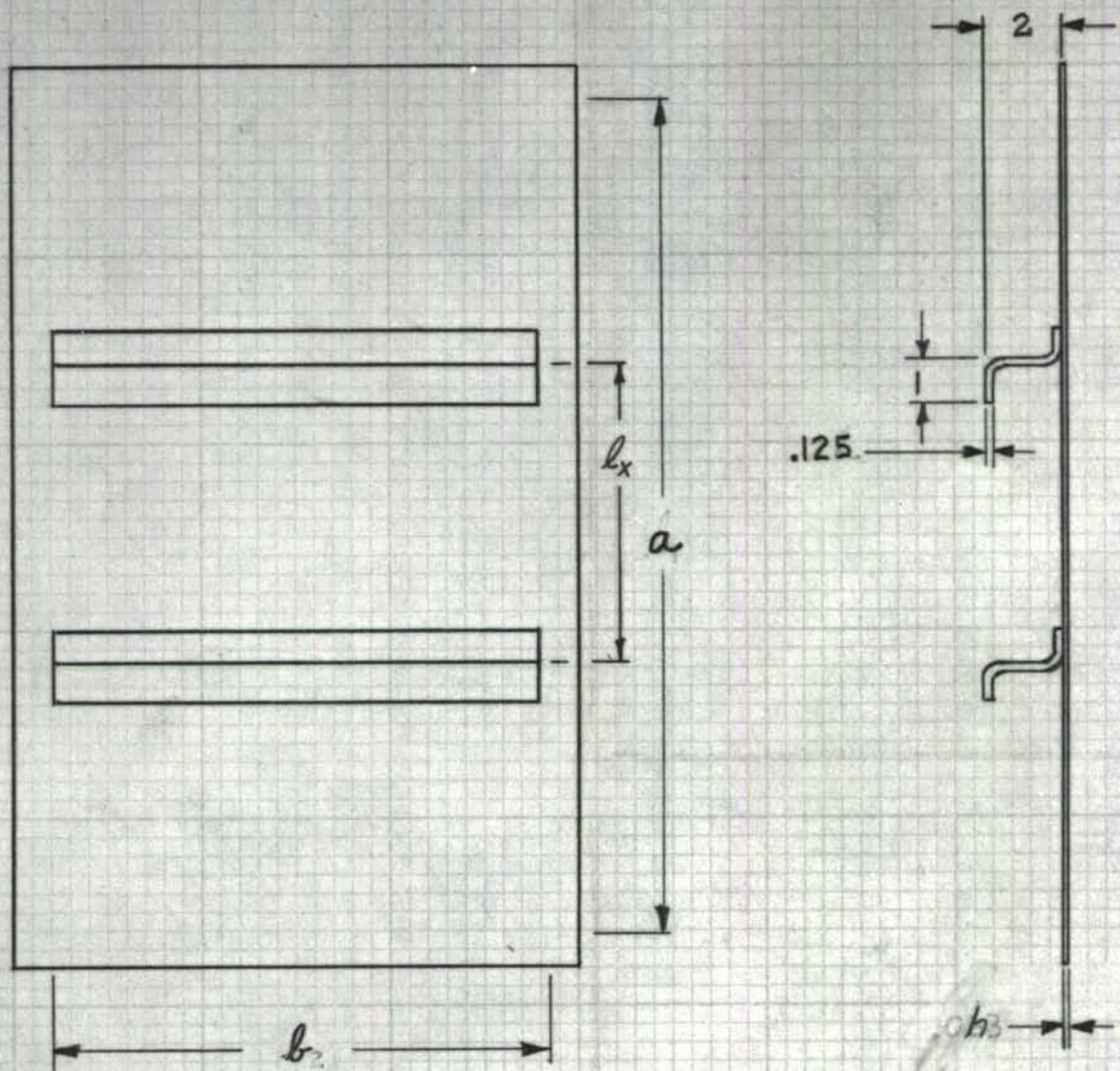
FIGURE 1. THREE BAY PANEL DESIGN
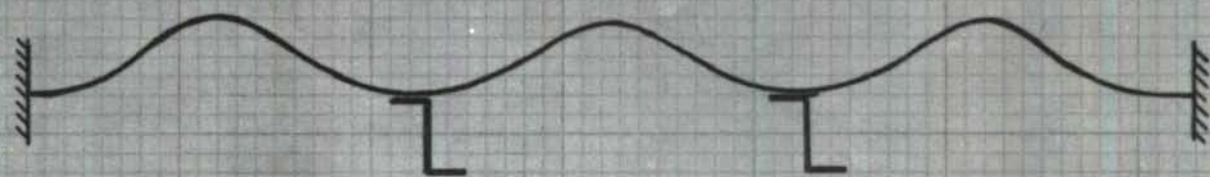
FIGURE 2. THREE BAY PANEL MODE SHAPES

this, the mode of Fig.   2a is assumed to be the critical mode in fatigue and is the only mode considered in this study.

Modal Properties and Response

The equation for the mode of interest (Figure   2a) will be assumed to be:

$$\Psi = (1 - \cos 6\pi x/a)(1 - \cos 2\pi y/b)$$

where
Ψ = mode shape

x = distance along length of panel

y = distance along width of panel

a = effective length of panel

b = effective width of panel

The remaining modal properties can be obtained from the expressions for kinetic energy (T) and potential energy (V) as shown below (Ref 1 & 2).

$$2T = \int_0^a dx \int_0^b dy\, \rho'(\partial w/\partial t)^2 \tag{1}$$

$$2V = D\int_0^a dx \int_0^b dy\, (\partial^2 w/\partial x^2 + \partial^2 w/\partial y^2)^2$$
$$- 2(1-\nu)\left[(\partial^2 w/\partial x^2)(\partial^2 w/\partial y^2)-(\partial^2 w/\partial x\,\partial y)^2\right] \tag{2}$$

where

ρ' = mass/unit area

$D = Eh^3/12(1-\nu^2)$

E = Young's modulus

h = thickness

ν = Poisson's ratio

w = displacement normal to the surface

$\xi(t)$ Modal amplitude

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Assuming that

$$w = \zeta(t) \left[1 - \cos(6\pi x/a)\right]\left[1 - \cos(2\pi y/b)\right]$$

equations 1 and 2 can be expressed as

$$2T = \rho' \dot{\zeta}^2 \int_0^a dx \int_0^b dy \left[1 - \cos(6\pi x/a)\right]^2$$
$$\left[1 - \cos(2\pi y/b)\right]^2 \tag{3}$$

$$2V = \zeta^2 D \int_0^a dx \int_0^b dy \left[(36\pi^2/a^2)\cos(6\pi x/a)\right.$$
$$\left.(1 - \cos(2\pi y/b) + (4\pi^2/b^2)\cos(2\pi y/b)(1 - \cos 6\pi x/a)\right]^2 - 2(1-\nu)$$

$$\left\{(36\pi^2/a^2)(4\pi^2/b^2)\cos(2\pi y/b)\cos(6\pi x/a)\right.$$
$$\left[1 - \cos(2\pi y/b)\right]\left[1 - \cos(6\pi x/a)\right] - (6\pi/a)\cdot(2\pi/b)\sin(2\pi y/b)$$
$$\left.\sin(6\pi x/a)^2\right\} \tag{4}$$

Changing the variables to:

$$\theta = 6\pi x/a \quad \text{and} \quad \phi = 2\pi y/b$$

$$d\theta = 6\pi/a \; dx \quad \text{and} \quad d\phi = 2\pi/b \; dy$$

and using: $\theta = 6\pi$ when $x = a$, $\phi = 2\pi$ when $y = b$; equations 3 and 4 become:

$$2T = \dot{\zeta}^2 \rho' \int_0^{6\pi} d\theta \int_0^{2\pi} d\phi \; (1 - \cos\theta)^2 (1 - \cos\phi)^2 (a/6\pi)(b/2\pi)$$

or

$$2T = \dot{\zeta}^2 \rho' (ab/12\pi^2) \int_0^{6\pi} (1 - \cos\theta)^2 d\theta \int_0^{2\pi} (1 - \cos\phi)^2 d\phi \tag{5}$$

and

$$2V = \zeta^2 D \int_0^{6\pi} d\theta \int_0^{2\pi} d\phi \left\{\left[(36\pi^2/a^2)\cos\theta(1-\cos\phi) + \right.\right.$$
$$\left.(4\pi^2/b^2)\cos\phi(1-\cos\theta)\right]^2$$
$$- 2(1-\nu)\left[(36\pi^2/a^2)(4\pi^2/b^2)\cos\phi\cos\theta(1-\cos\phi)\cdot\right.$$
$$\left.\left.(1-\cos\theta) - (6\pi/a)(2\pi/b)\sin\phi\sin\theta)^2\right]\right\}\frac{a}{6\pi}\frac{b}{2\pi}$$

or

$$2V = \dot{\xi}^2 \, 12 \, ab\pi^2 \, D \int_0^{6\pi} d\theta \int_0^{2\pi} d\phi \left[9/a^4 (1 - \cos \phi)^2 \right.$$

$$\cos^2\theta + 1/9 \, b^4 (1 - \cos \theta)^2 \cos^2\phi +$$

$$(2\nu/a^2 b^2)\cos \theta \cos \phi (1 - \cos \theta)(1 - \cos \phi) +$$

$$\left. \{2(1-\nu)/a^2 b^2\} \sin^2\theta \sin^2\phi \right] \tag{6}$$

Equations 5 and 6 are solved by means of the following integral forms:

$$\int_0^{m\pi} \cos^2 z \, dz = \int_0^{m\pi} \sin^2 z \, dz = m\pi/2$$

$$\int_0^{m\pi} (1 - \cos z) \cos z \, dz = -m\pi/2$$

$$\int_0^{m\pi} (1 - \cos z)^2 \, dz = 3m\pi/2$$

Therefore,

$$2T = (9/4) \, \rho' \, a \, b \, \dot{\xi}^2 \tag{7}$$

and

$$2V = \dot{\xi}^2 \, 12 \, ab\pi^2 \, D \, (3\pi^2) \, (R) \tag{8}$$

where

$$R = (27/a^4 + 1/3b^4 + 2/a^2 b^2)$$

It can be shown that:

$$2T = M \, \dot{\xi}^2$$

and

$$2V = K \, \xi^2$$

where

M = Modal mass

K = Modal stiffness

This results in the frequency for the mode of interest being:

$$\omega^2 = K/M = 16\pi^4 \, D/\rho' a^2 b^2 \left[27 \, (b/a)^2 + 1/3 \, (a/b)^2 + 2 \right] \tag{9}$$

Letting

$$E = 10^7 \text{ psi}$$

$$\rho' = \rho h \quad (.1 \, h/386) \, (\text{lb sec}^2/\text{in}^4)$$

$$\nu = .3$$

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$$a = 24 \text{ in}$$

$$b = 13 \text{ in}$$

gives

$$\omega = 2.52 \times 10^4 \text{ (h/in) (radians/sec)}$$

or

$$f = 4.05 \times 10^3 \text{ (h/in) (cycles/sec)}$$

The stress of interest is the maximum stress on the panel. This will occur in the outer fibers and at the center of the long boundary (stiffeners). The expression for the stress is (Ref. 1):

$$S_x = -E(h/2)(1 - \nu^2)\left[(\partial^2 w /\partial x^2) + \nu(\partial^2 w / \partial y^2)\right] \tag{10}$$

where

$$w = A_m(1 - \cos 6\pi x/a)(1 - \cos 2\pi y/b)$$

w = displacement normal to the plate

A_m = the amplitude of the mode

This gives:

$$S_x = EA_m(\pi^2/2)(1 - \nu^2)\left[(36/a^2)\cos(6\pi x/a)(1 - \cos 2\pi y/b)\right.$$
$$\left. + \nu 4/b^2 \cos(2\pi y/b)(1 - \cos 6\pi x/a)\right]$$

The maximum stress occurs at:

$$x = 0, \ a/3, \ 2a/3 \text{ and } a \ ; \ y = b/2$$

Therefore,

$$S_x = 36 \ EA_m h \pi^2/a^2 (1 - \nu^2) = 12 \cdot 36 \pi^2 \ A_m D/h^2 a^2 \tag{11}$$

For the static case, A_m is the static modal deflection, or

$$A_m = \frac{\text{Modal force due to unit pressure}}{\text{Modal stiffness}} = \frac{F_s}{K}$$

and

$$F_s = \int_0^a dx \int_0^b dy \ [1 - \cos (6\pi x/a)][1 - \cos (2\pi y/b)] = ab$$

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This gives

$$A_m = ab/K \tag{12}$$

where

$$K = 36 \pi^4 \, Dab \, (27/a^2 + 1/3b^4 + 2/a^2b^2)$$

as previously shown.

Substituting Eq. 12 into Eq. 11 results in the maximum static stress per unit static pressure as:

$$S_s/P_s = 18.4/h^2$$

The dynamic stress can be expressed as (Ref. 3):

$$S_{RMS} = (\pi/2 \ Q \ f \ PSD \ (f))^{1/2} \ S_s/P_s \tag{13}$$

where

S_{RMS} = Maximum RMS stress

Q = Quality Factor

f = Natural frequency

$PSD(f)$ = Power spectral density of the exciting pressure

S_s/P_s = Static stress/static pressure

The acoustic energy for the test will be limited to a band width of 3 times the frequency band width between half power points of the response mode of interest, or:

$$\Delta f = 3f/Q$$

The power spectral density of the acoustic energy in this band can be expressed as:

$$PSD(f) = I_a/\Delta f = I_a Q/3f \tag{14}$$

where

I_a = square of the acoustic pressure acting on the panel.

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Combining equations 13 and 14 gives:

$$S_{RMS} = (\pi I_a/6)^{1/2} Q \, S_s/P_s \tag{15}$$

Assuming that the Q of the panel is 10, and the level obtainable in the WPAFB fatigue facility is 162 db (.365 psi) yields:

$$S_{rms} = 50.0/h^2$$

The desired stress is the random fatigue strength for 7075-T6 aluminum at 10^5 cycles. This value is slightly less than 20,000 psi as obtained from Figure 3. The resulting thickness is:

$$h^2 = 50/20,000 = 25 \times 10^{-4} \, in^2$$

or $h = 0.05$ in

This value is the result of a conservative analysis. For example a stress concentrative factor of 1.25, or a Q of 12.5, or an SPL of 164 db would raise the value to:

$h = 0.0625$ inches

The stress concentration factor of 1.25 is more realistic. The Q of 12.5 is probably still conservative, and the 164 db is probably obtainable. Therefore, a thickness of 0.0625 inches is still conservative and will be used in the remainder of this section.

Effect of Stiffeners

The Z section stiffeners will not be sized but rather will be assumed to be 1 in x 2 in x 1 in x 0.125 in. (The thickness is 2h). This portion of the analysis utilizes the simplified models given in Fig. 4 and shows that Z section flexibility will not invalidate the previous analysis.
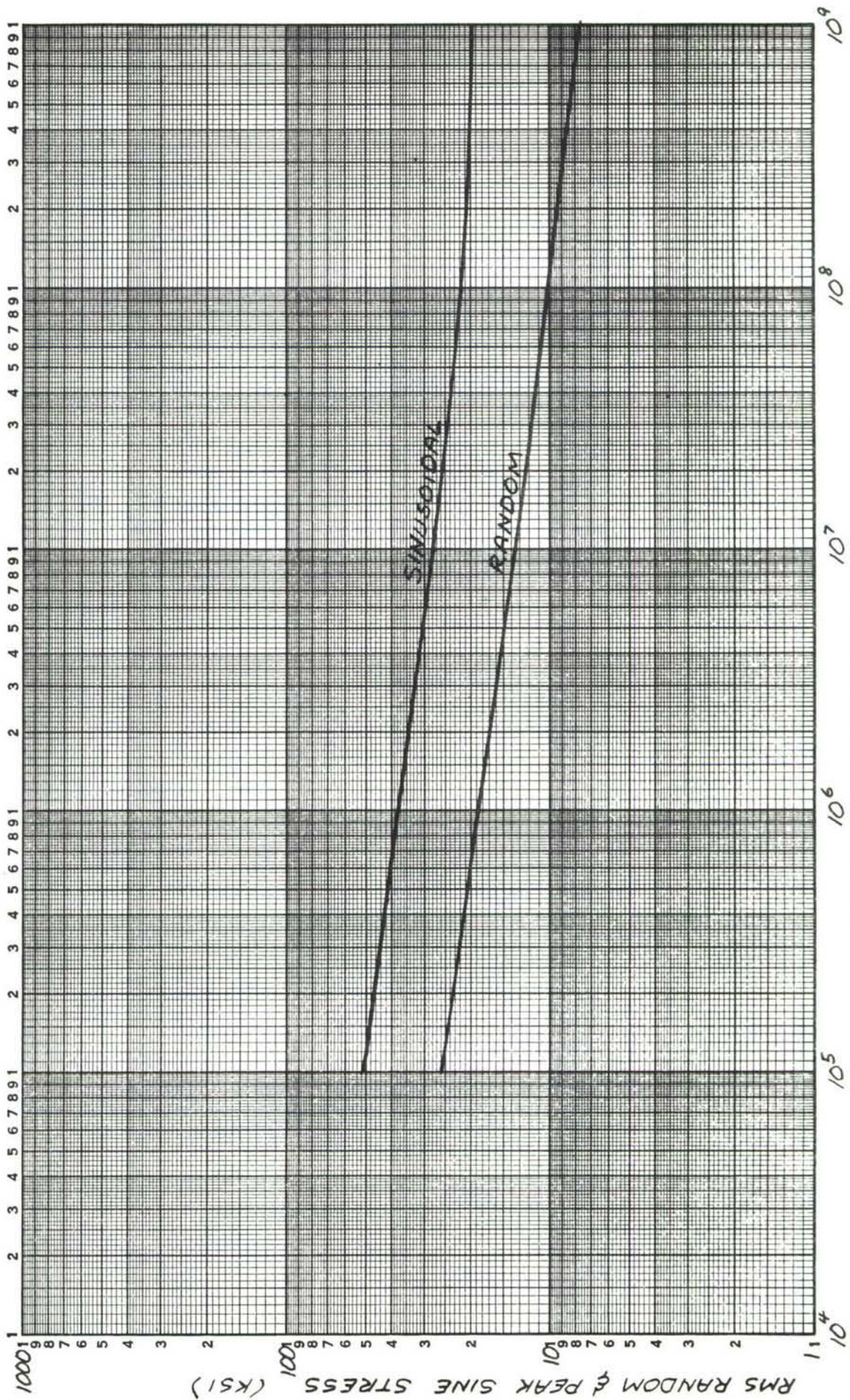
FIGURE 3   SINUSOIDAL AND RANDOM FATIGUE DATA FOR 7075-T6 ALUMINUM ALLOY (REFERENCES 3 & 4)
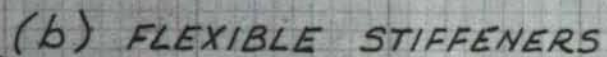
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(a) RIGID STIFFENERS

(b) FLEXIBLE STIFFENERS
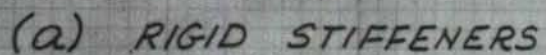
FIGURE 4. DYNAMIC MODEL OF THREE BAY PANEL

First, some of the properties of the Z section must be defined.

Its mode shape will be assumed to be:

$$\Psi = \xi(1 - \cos 2\pi y/b)$$

The modal properties will be obtained from the kinetic energy and frequency:

$$2T = m \int_{0}^{b} (\partial w/\partial t)^2 \, dy \qquad (16)$$

$$m = 0.50\rho = \text{mass/unit length}$$

for thickness = 0.125 inches (2h).

$$2T = .50\rho \, \dot{\xi}^2 \int_{0}^{b} (1 - \cos 2\pi y/b)^2 \, dy$$
$$= \dot{\xi}^2 \, .50\rho b/2\pi \int_{0}^{2\pi} (1 - \cos \theta)^2 \, d\theta$$
$$= .50 \, \dot{\xi}^2 \, b\rho \, 3\pi/2\pi = \dot{\xi}^2 \rho \, b \, .75$$

Therefore the modal mass is:

$$M = .75\rho b = 2.52 \times 10^{-3} \text{ lbs sec}^2/\text{in}$$

From standard frequency tables (Reference 5)

$$f = 2.87 \times 10^3$$

$$\omega = 1.80 \times 10^4$$

$$\omega^2 = 3.32 \times 10^8$$

and

$$K = M\omega^2 = 8.36 \times 10^5 \text{ lb/in}$$

The model shown in Figure 4a represents the plate pinned by the stiffeners and is compatible with the mode shape of interest. This model will be solved in order to relate the k and M values to the previous plate model.

A solution can be obtained for the model using the potential and kinetic energies as shown below:

$$2T = 3M \, \dot{X}_1^2$$

$$2V = k_1 \left[4 (\theta/2)^2 + 5\theta_1^2 \right] = 6k_1 \theta_1^2$$

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where $\qquad$ $M = 1/3$ of the plate's modal mass

$\qquad k_1 =$ spring constant required to give the proper stiffness.

Assuming small displacement

$\qquad \theta_1 = 2x_1/L \qquad$ where $L = a/6$

and $\qquad 2V = 24k_1 \, x_1^2/L^2$

let $\qquad k = k_1/L^2$

therefore:

$\qquad 2V = 24k x_1^2$

and $\qquad \Omega_1^2 = 8k/M$

Ω_1^2 is the frequency previously found for the plate. This results in

$\qquad k = \Omega_1^2 M/8$

The model shown in Fig. 4b represents the plate supported by flexible Z sections. The mass and stiffnesses of the Z sections (M_2 and k_2) are the modal mass and stiffness previously found. The solution for the model can be obtained as shown below.

$$2T = M_1 (2 \, \dot{x}_1^2 + \dot{x}_3^2) + 2M_2 \, \dot{x}_2^2 \qquad\qquad (19)$$

$$2V = k_1 (4\theta_1^2 + 2\theta_2^2 + 2\theta_3^2 + \theta_4^2) + 2 k_2 x_2^2 \qquad\qquad (20)$$

where

$\qquad \theta_1 = x_1/L \, , \qquad \theta_2 = (x_2 - x_1)/L - x_1/L \, ,$

$\qquad \theta_2 = (x_2 - 2x_1)/L \, , \qquad \theta_3 = (x_3 - x_2)/L - (x_2 - x_1)/L$

$\qquad \theta_3 = (x_3 - 2x_2 + x_1)/L \, ,$

$\qquad \theta_4 = 2 \, (x_2 - x_3)/L$

therefore

$$2V = k \left[4x_1^2 + 2(x_2 - 2x_1)^2 + 2 (x_3 - 2x_2 \quad x_1)^2 + 4(x_2 - x_3)^2 \right] + 2 k_2 x_2^2$$

It can be shown that the potential and kinetic energies can be put into the form:

$$T = 1/2 \sum_{i=1}^{n} \sum_{J=1}^{n} \dot{x}_i \dot{x}_j M_{ij}$$

$$V = 1/2 \sum_{i=1}^{n} \sum_{J=1}^{n} x_i x_j K_{ij}$$

where $n =$ number of degrees of freedom and, M_{ij} and K_{ij} are the elements of the mass and stiffness matrices.

The frequencies and mode shapes can be obtained from the solution of $\left[K - \omega^2 M \right] = 0$ which is the standard eigen-value problem.

Therefore,

$$M = \begin{bmatrix} 2M_1 & & \\ & 2M_2 & \\ & & M_1 \end{bmatrix}$$

and

$$K = \begin{bmatrix} 14\,k & -8\,k & 2\,k \\ -8\,k & 14\,k & 2\,k - 8\,k \\ 2\,k - 8\,k & & 6\,k \end{bmatrix}$$

and

$$0 = \begin{bmatrix} 14\,k - 2M_1\omega^2 & -8\,k & 2\,k \\ -8\,k & (14\,k \quad 2\,k_2 - 2\omega^2 M_2) & -8\,k \\ 2\,k & -8\,k & 6\,k - {}^2 M_1 \end{bmatrix}$$

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On expansion, the determinant becomes

$$\left[\omega^6 - (13M_2M_1 k_1 + M_1^2 k_2 + 7 k_1 M^2)\omega^4 + \right.$$
$$(40 M_2 k_1^2 + 13 M_1 k_1 k_2 + 43 M_1 k_1^2)\omega^2$$
$$\left. - (24 k_1^3 + 40 k_1^2 k_2)\right] \quad 1/M_2 M_1^2 = 0 \tag{21}$$

Recall that $\Omega_1^2 = 8k_1/M_1$

and

$$\Omega_2^2 = k_2/M_2 = \text{squared circular frequency of the stiffener}$$

Let

$$S = \omega^2/\Omega_1^2 \quad , \quad \Omega_2^2/\Omega_1^2 = k_2 M_1/8k_1 M_2$$

where ω = natural frequency of the system, and rewrite equation 21; this yields:

$$S^3 - (13/8 + 7/8 \ M_1/M_2 + \Omega_2^2/\Omega_1^2) S^2 + (43/64 \ M_1/M_2 + 13/8 \ \Omega_2^2/\Omega_1^2 + 5/8)S$$
$$- (3/64 \ M_1/M_2 + 5/8 \ \Omega_2^2/\Omega_1^2) = 0$$

The numerical values are:

$$\Omega_2^2/\Omega_1^2 = 145 \text{ and } M_1/M_2 = 1.5$$

This gives

$$S^3 - 148 \ S^2 + 238 \ S - 91 = 0$$

The root of interest should be the first one less than $S = 1$. Assuming this
is the case, an approximate value can be obtained from:

$$S^2 - S \ 238/148 + 91/148 = 0 \quad \text{or}$$
$$S = \left[1.61 \pm (0.14)^{1/2}\right]/2$$
$$S = 0.99 \text{ and } 0.62$$

Using 1 as a first approximation and applying Newton's Method which states:

$$x_2 = x_1 - f(x)/f'(x)$$

gives:

$$S = 1.00 - (1 - 148 + 238 - 91)/(3 - 296 + 238)$$

$$S = 1 - 0 = 1$$

The mode shapes can be obtained from any two of the equations of motion derived from the energy expressions:

$$\partial/\partial t \left[\partial (T-V)/\partial \dot{x}_i \right] - \partial(T-V)/\partial x_i = 0$$

The two chosen are given below:

$$-\omega^2 2M_1 x_1 + k_1 (4x_1 + 8x_1 - 4x_2 + 2x_1 - 4x_2 + 2x_3) = 0$$

$$-\omega^2 M_1 x_3 + k (2x_3 - 4x_2 + 2x_1 + 4x_3 - 4x_2) = 0$$

Putting these in terms of S and rearranging and letting $x_1 = 1$ gives:

$$(-S8 + 7) - 4x_2 + x_3 \qquad = 0$$

$$+2 \qquad - 8x_2 + x_3 (-S8 + 6) = 0$$

or

$$x_3 = (16S - 12)/(8S - 4) = 4S - 3/2S - 1 \text{, for } S = 1$$

$$x_3 = (16 - 12)/(8 - 4) = 1$$

and

$$x_2 = (8S - 7 - 1)/-4 = 0$$

Therefore, within the accuracy of this model, the stiffeners will
not affect the sizing analysis. It is concluded that the plate shown in
Figure 1, if constructed of 0.0625 in. 7075-T6 aluminum, stiffened by
0.125" x 2" x 1" Z section stiffener will fail in less than 10^5 cycles when
subjected to the acoustic field assumed previously.

2.2 LINEARITY ANALYSIS

Non-linearity of the panel behavior due to membrane stresses could
affect the fatigue life of the panel. This effect is examined for one bay of the
three bay panel.

The membrane stress can be determined from the change in length
of a strip of unit width across the panel. The cross section of the panel
is shown in Figure 2.5. From this figure the differential increment of width
across the panel can be expressed as

$$ds = dx/\cos \theta$$
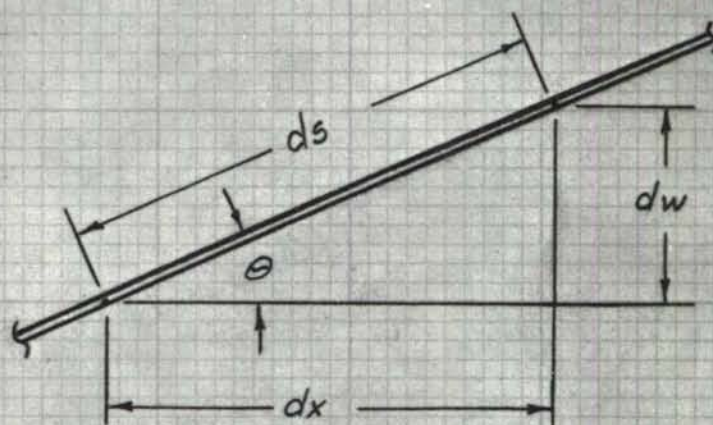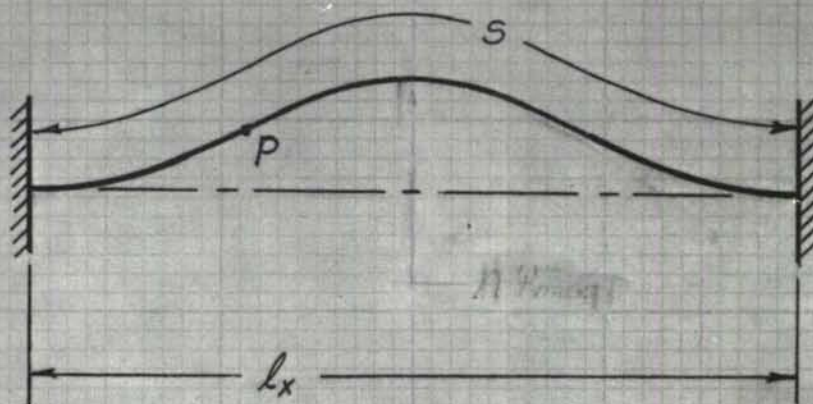
The overall change in length is:

$$\Delta \ell_x = S - \ell_x = \int_0^{\ell_x} dx/\cos \theta - \int_0^{\ell_x} dx$$

or

$$\Delta \ell_x = \int_0^{\ell_x} (1 - \cos \theta)/\cos \theta \ dx$$

Since θ is small, $\cos \theta \simeq 1$,

$$\Delta \ell_x = \int_0^{\ell_x} (1 - \cos \theta) \ dx$$

FIGURE 5. MODE SHAPE AND SYMBOLS
FOR LINEARITY ANALYSIS

and

$$\cos \theta \cong 1 - \theta^2/2 + \ldots$$

Therefore

$$\Delta\ell_x = \int_0^{\ell_x} \theta^2/2 \; dx$$

A mode shape is assumed as follows,

$$\Psi(x,y) = (1/4)(1 - \cos 2\pi x/\ell_x)(1 - \cos 2\pi y/b)$$

for which the displacement is

$$w(x,y) = \xi/4 \; (1 - \cos 2\pi x/\ell_x)(1 - \cos 2\pi y/b)$$

From Figure 5 it can be seen that

$$\theta = dw/dx$$

At the point of maximum displacement (where $y = b/2$),

$$\theta = \xi\pi/\ell_x \sin(2\pi x/\ell_x)$$

and

$$\Delta\ell_x/\ell_x = \xi^2\pi^2/(4\ell_x^2)$$

The definition of Young's modulus is:

$$E = \sigma/\Delta\ell/\ell$$

Therefore

$$\sigma_T = E \; \xi^2\pi^2/(4\ell_x^2)$$

The membrane stress, σ_T, is proportional to ξ^2; the bending stress, σ_B, proportional to ξ. Therefore the σ_T as a function of σ_B can be defined if both are evaluated at the same ξ.

The bending stress is expressed as:

$$\sigma_B = (E/1-\nu^2)(h/2)(\partial^2 w/\partial x^2 + \nu \partial^2 w/\partial y^2)$$

Evaluating the expression at the point of max. bending stress, $x = 0$ and $y = b/2$ gives

$$\partial^2 w/\partial x^2 = \xi 2\pi^2/\ell_x^2$$

and

$$\partial^2 w/\partial y^2 = 0$$

The physical constants are

$$E = 10^7 \text{ psi}$$

$$\nu = .3$$

$$\sigma_B = 20,000 \text{ psi}$$

$$h = 0.0625 \text{ IN.}$$

$$\ell_x = 8 \text{ in}$$

Giving

$$\xi = .188 \text{ in}$$
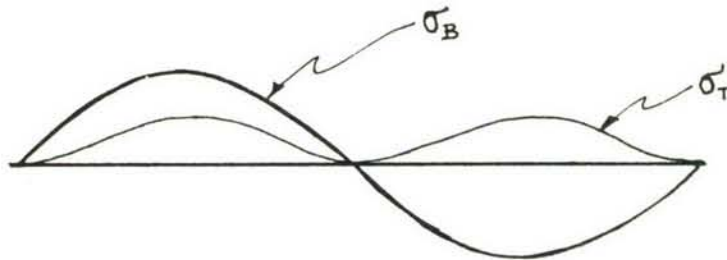
Resulting in a membrane stress of

$$\sigma_T = 13,600 \text{ psi.}$$

Before any conclusions can be obtained from this result the effects of the non-linearity on fatigue life must be determined.

For one complete cycle of the panel, the tensile stress is zero when the bending stress is zero, and is maximum positive when the bending stress is maximum positive or negative as shown below:

The net effect is similar to a preload having a magnitude equal to the maximum of the tensile stress. Assuming this is the case, the required information can be obtained from a constant life chart (Ref. 2) with σ_T' as a function of σ_B' superimposed, as shown in Figure 6.

The sinusoidal fatigue curve can be obtained by plotting the fatigue stress, as a function of life, along the zero mean stress curve. In a like manner, the effect of the non-linearity on the fatigue can be accounted for by plotting the intersections of the constant life line and the curve of σ_T' vs σ_B'. Such S-N curves are shown in Fig. 7.

Assuming a Rayleigh peak distribution for σ_B', the random fatigue curves can be obtained using the method described in Ref. 4. This results in the random curves shown in Figure 7. The use of a Rayleigh peak distribution for the non-linear analysis does not give exact results because the panel response is amplitude sensitive, thereby reducing the higher amplitude peaks that are predicted from a Rayleigh distribution. If such effects were accounted for in the analysis, it would result in the non-linear random fatigue curve shown in Figure 7 being rotated in the clockwise direction. Such an analysis however is beyond the scope of this study.
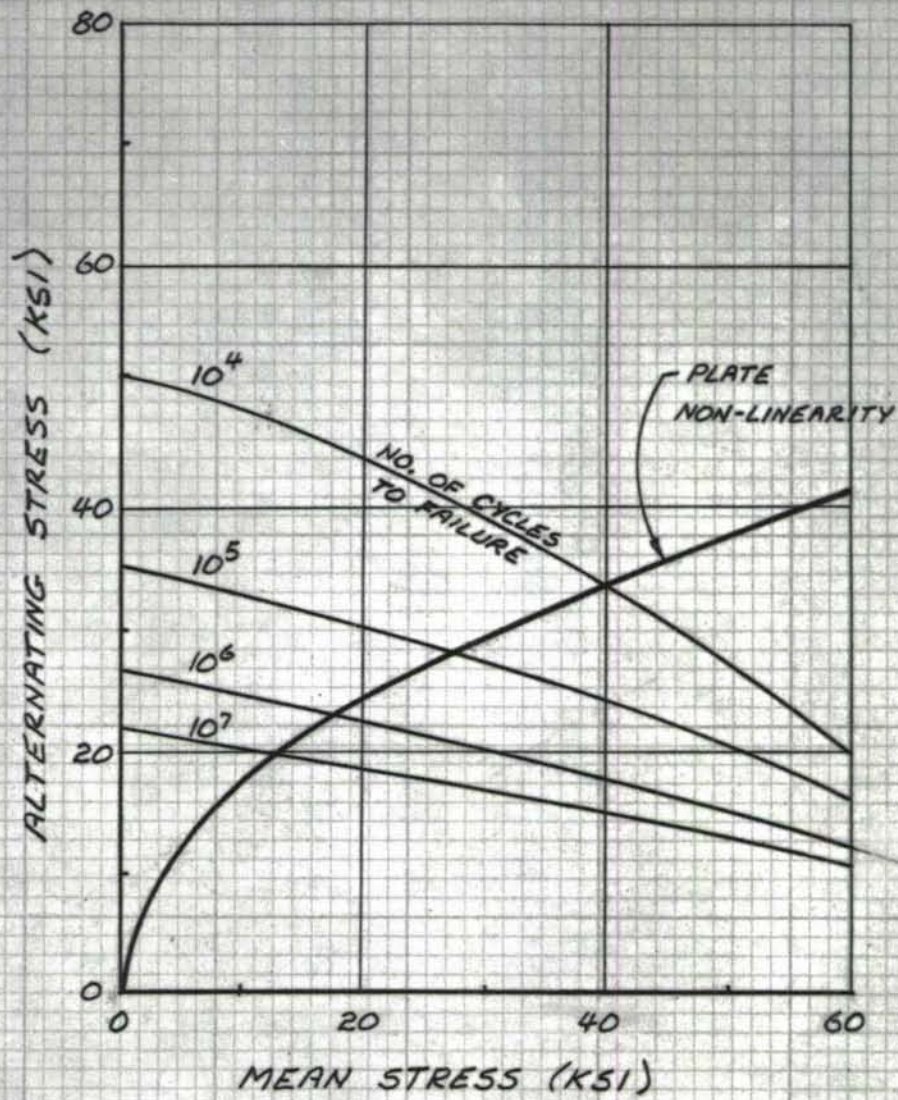
FIGURE 6. CONSTANT LIFE CHART
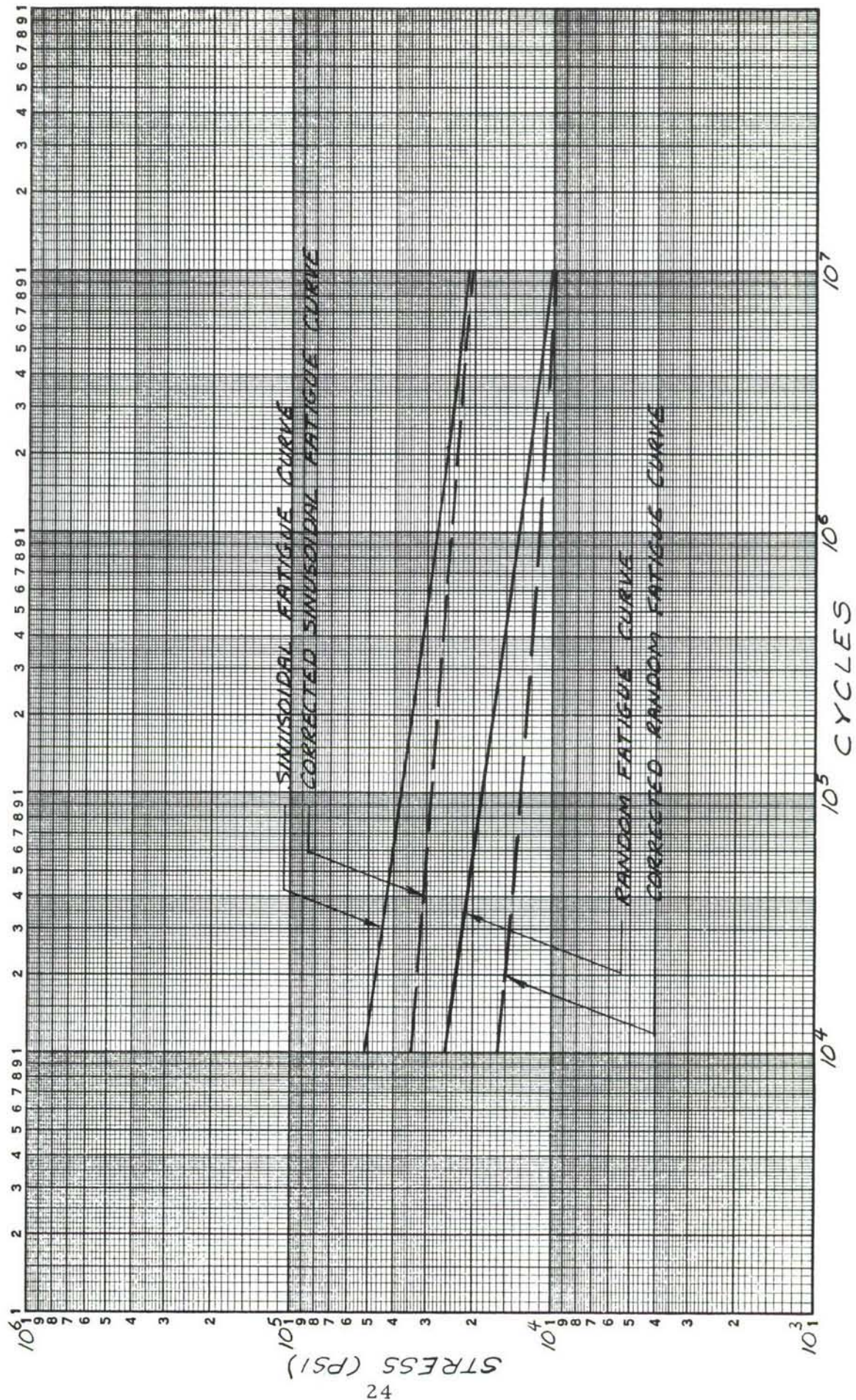FOR 7075-T6 ALUMINUM

FIGURE 7. FATIGUE CURVES FOR 7075-T6 ALUMINUM

SINUSOIDAL FATIGUE CURVE
CORRECTED SINUSOIDAL FATIGUE CURVE

RANDOM FATIGUE CURVE
CORRECTED RANDOM FATIGUE CURVE

STRESS (PSI)

CYCLES

3. DESIGN OF FRAME-STRINGER PANELS

3.1 INTRODUCTION

This section describes the detail design of the frame-stringer panels. Three panel parameters are considered: static pressure, stringer cross section, and panel curvature. Six variations of the design are presented as shown in the following table:

Design No.	Aerodynamic Pressure psig	Internal Pressure psig	Stringer Type	Curvature
1	2	10	J	Flat
2	4	15	J	Flat
3	2	10	Hat	Flat
4	4	15	Hat	Flat
5	2	10	J	60"R
6	2	10	Hat	60"R

All panels are 48 inches wide by 72 inches long, with lengthwise Z frames spaced at 9 inch intervals, and stringers spaced across the full width of the panels at 5 inch intervals. Designs of curved and flat panels are based on loads produced in a cylindrical pressurized shell ten feet in diameter. The section properties for stringers apply to both J-sections and hat sections; thus, only two analyses are required, - one for each of two loading conditions. The analysis is presented in detail for the 10 psig internal pressure condition. The 15 psig analysis is summarized for each structure component and all results presented. They are referred to as the "10 psi design" and the "15 psi design."

3.2 DESIGN CONDITIONS

All panels have skin thickness, stringers and Z frames designed to resist tension loads perpendicular and parallel to the frames. These loads are to be equivalent to loads produced in a cylindrical pressurized shell ten feet in diameter. Two pressure loadings are considered: first, a 10 psig internal burst pressure , with a 2 psig aerodynamic pressure normal to the skin, used for designs 1, 3, 5 and 6 listed in the previous table; similarly a 15 psig internal burst pressure, and a 4 psig aerodynamic pressure, used for designs 2 and 4.

The analysis of the skin is based on a single 5 x 9 inch panel with an internal static pressure, and an external uniformly distributed aerodynamic pressure P_a which is either static, or which varies from p_a to $-p_a$ psig at a rate sufficiently slow such that the structure responds statically to the applied load.

The design conditions are summarized as follows:

1. A long cylindrical shell, ten feet in diameter.

2. Internal burst pressure 10 psig or 15 psig (two independent conditions.)

3. External aerodynamic pressure, 2 psig or 4 psig, additive to 10 psig or 15 psig internal pressure respectively.

4. Skin material is 7075-T6 aluminum. Design stress of the material equal to ultimate stress which is 72,000 psi.

5. Stringers designed for bending and tension loads.

3.3 DESIGN OF SKIN

If the cylinder had no frames or stringers, the skin would be in tension due to internal pressure. With a framed shell, however, the rigid frames take out the tangential loads and the skin loading reduces to the normal

pressure and the boundary restraints of the frames and stringers. For this analysis the skin is designed to resist normal pressure only. The stringers are designed to limit their center-span displacement to approximately 10^{-3} inches relative to the frames. Finally, the frames are designed to exceed the support strength required for the panel edge reactions and the stringers; the sum of these loads being the total radial force developed.

3.3.1 Normal Pressure

Boundary conditions for the entire frame-stringer panel are not to be considered in this analysis. Therefore, the design of the skin reduces to a consideration of a single 5 x 9 inch panel with an internal burst pressure plus a superimposed external aerodynamic pressure.

Since the arc width of the panel is only five inches, with a radius of curvature of sixty inches, the panel strength may be determined approximately by a flat plate analysis (Wang, Reference 5). The panel is supported on either side by stringers and on the ends by frames. The frames are rigid with respect to the sheet. The adjacent panels provide tension and bending restraint at the sides along the stringers. The stringers can deflect normal to the skin; but for the present they are considered rigid.

3.3.2 Linear Deflection Theory

Timoshenko (Reference 1) gives the equations for a fixed edge thin ($w/t > .5$) panel with ratio of sides $a/b = 1.8$, as:

$M_x = 0.0812 \ pb^2$, at center of long side

$w_x = 0.0267 \ pb^4/Et^3$, at center of panel

where $p =$ normal pressure, psi

$b =$ width of panel, in.

$t =$ thickness of panel, in.

$E =$ Young's modulus, psi

Solving for the bending moment at the center of a long side:

$$M_x = 0.0812 \ (12) \ (5)^2 = 24.35 \ \text{lb in/in}$$

We now determine the minimum thickness allowed: The section modulus is

$$Z = I/C = bt^2/6$$

Substituting in the bending stress equation:

$$\sigma = M/Z = 24.35(6)/t^2 = 72,000 \ \text{psi}$$

Minimum thickness is:

$$t = 0.045 \ \text{inches}$$

The plate deflection may now be calculated:

$$w_x = 0.0267 \ \ pb^4/Et^3 = 0.0267 \ \ 12 \times (5)^4/10 \ (0.45)^3$$

$$w_x = 0.183 \ \text{in.}$$

The deflection/thickness ratio is $0.183/0.045 = 4.04$

Timoshenko, Reference 1, page 333, indicates a membrane type analysis should be used if the maximum deflection exceeds $t/2$. Therefore, we must consider the panel as a membrane. The 15 psig internal pressure condition also requires membrane theory analysis.

3.3.3 Large Deflection Theory

When the deflection of a panel becomes large the normal loads are taken out by stretching of the surface and by bending. These large deflections introduce nonlinear terms into the conditions of equilibrium and are governed by two fourth-order, second-degree, partial differential equations. For the case at hand, use is made of existing available data pertinent to the solution of these equations. References 5 through 9 treat the problem of large deflection in panels. There is general agreement as to results although the papers differ in their method of solution, boundary conditions, and panel dimensions.

Roark, Reference 10, presents a table of coefficients for large deflections of panels, under uniform load, with $a/b = 1.5$. The deflection and stress coefficients are plotted in Figures 8 and 9, respectively. The abscissa for both curves is the dimensionless parameter pb^4/Et^4, a function of the panel loading and thickness. In order to determine what panel thickness corresponds to the design stress, several t's are chosen and stresses computed. The results are tabulated below for the 10 psig internal pressure design. Results for both design conditions are plotted in Figure 10.

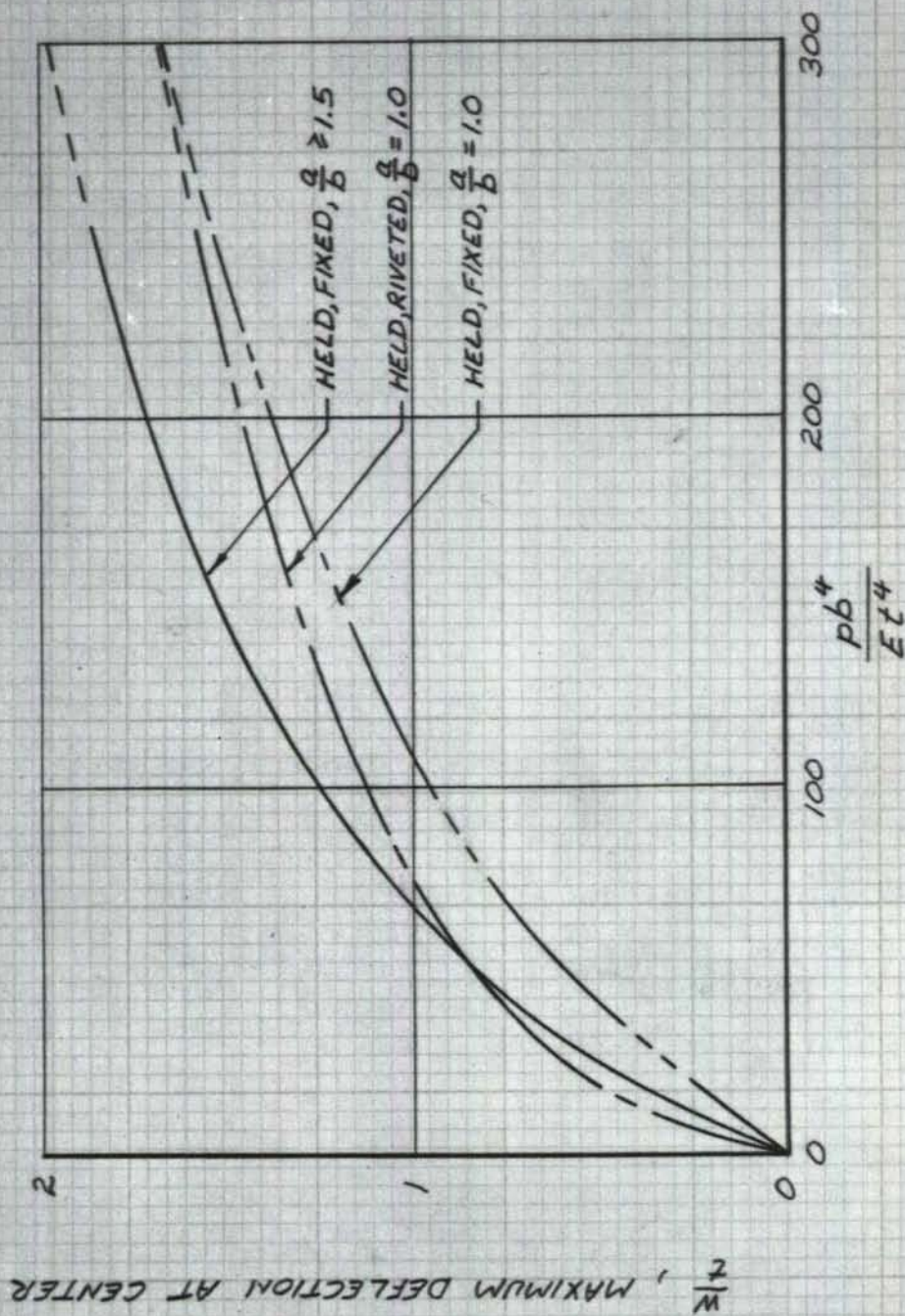| t	pb^4/Et^4	w/t	$\sigma b^2/Et^2$	w, in.	σ, psi
.025	1900	-	-	-	-
.032	714	3.06	155	.098	63,500
.040	293	1.96	76.5	.0785	49,000
.050	120	1.37	40.0	.0685	40,000
.063	48	0.81	19.7	.0510	31,200
.070	31	0.60	13.6	.0420	26,600

FIGURE 8. MAXIMUM DEFLECTION AT CENTER OF A RECTANGULAR
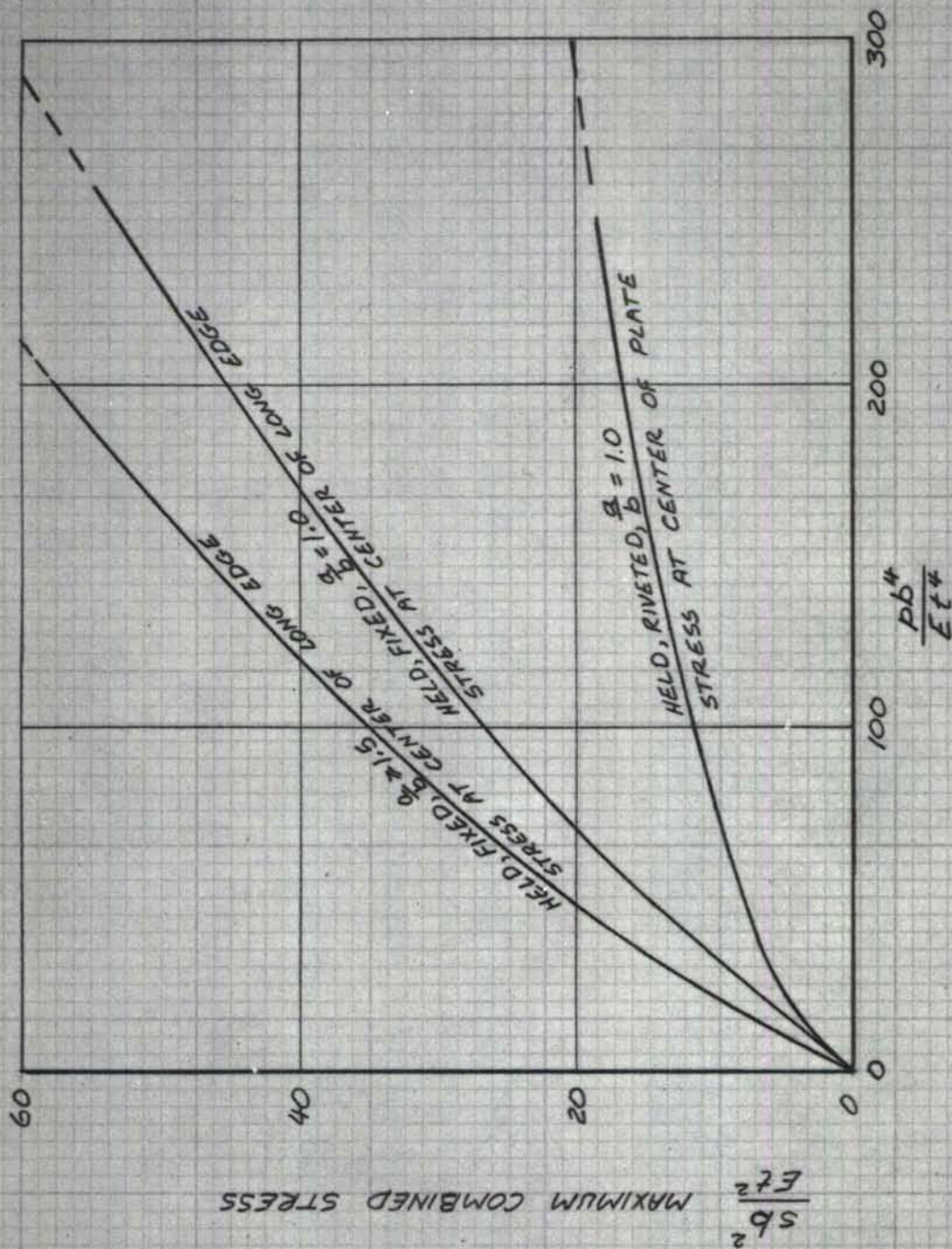
PLATE UNDER UNIFORM PRESSURE

FIGURE 9. MAXIMUM COMBINED STRESS IN A
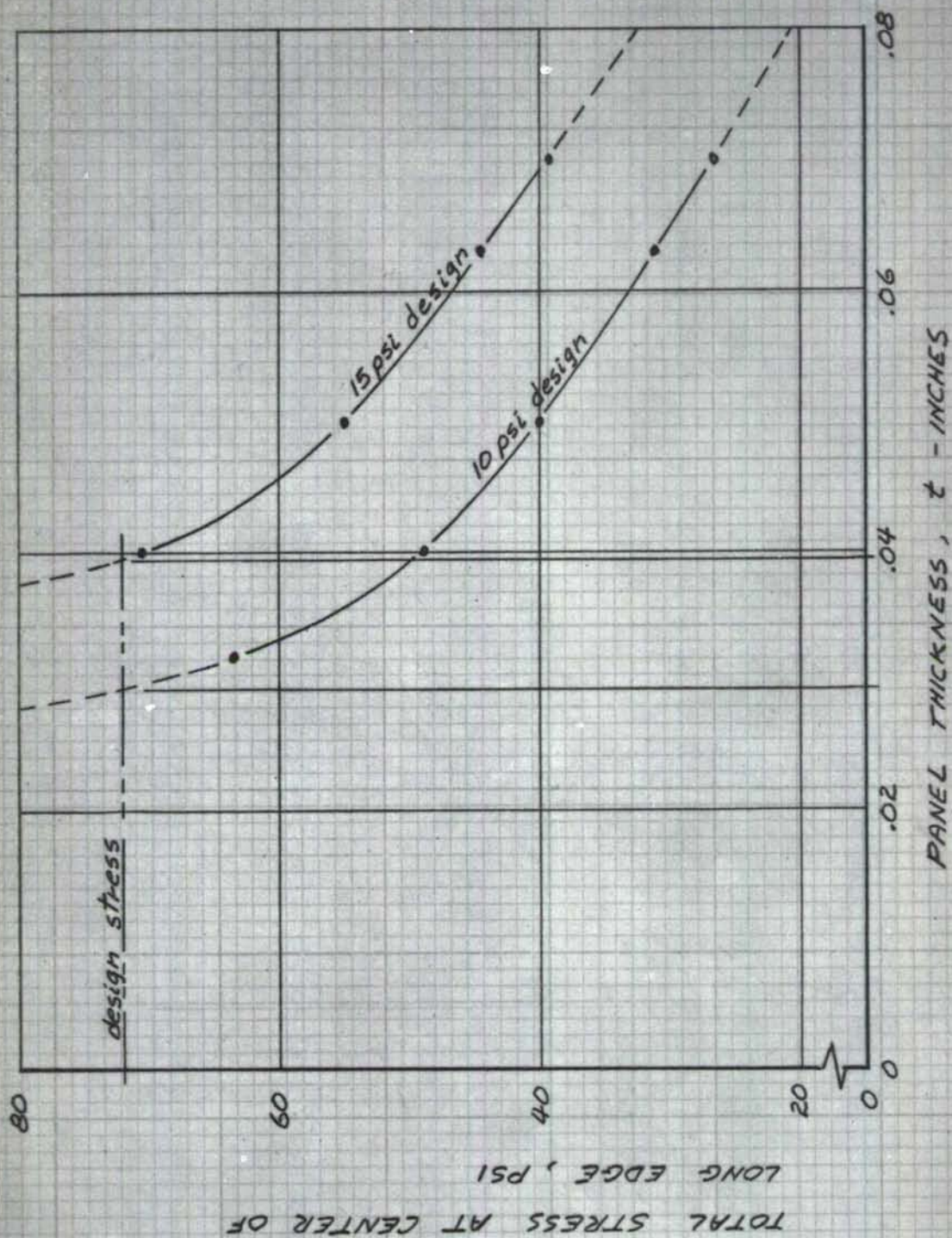
RECTANGULAR PLATE UNDER UNIFORM PRESSURE.

FIGURE 10. STRESS VS. THICKNESS FOR A FIXED PLATE WITH LARGE DEFLECTIONS, $\frac{a}{b} \geq 1.5$

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Based on this fixed edge analysis, the skin thickness should be 0.0295 inches and 0.0395 inches for the 10 psi and 15 psi designs, respectively. Selecting the nearest commercially available sheet, the skin thickness become 0.032 and 0.040 inches.

Thus far the 5 x 9 inch panel has been assumed to be fixed at the stringer centerline. Actually a double row of rivets is used, thus reducing the effective panel width. Once the stringer designs are set, the skin thickness selection is reviewed and modified if required, based on the effective panel width.

3.3.4 Effect of Stringer Deflection

The maximum stress in the panel occurs in bending at the center of the long edge. Deflection of the stringers occurs in a plane perpendicular to the panel and parallel to the long edge. Deflection in this plane would have a second order effect on stress in the plane normal to the long edge. We may, therefore, consider the fixed edge analysis above to be final, keeping in mind that stringer deflection must be minimized.

3.4 DESIGN OF STRINGERS

3.4.1 Axial Tension

Pressure on the ends of a closed cylinder produces axial loads in the structure. The total force is:

$$P = p(\text{Area}) = p \pi R^2$$

The number of stringers is

$$N = \text{circ.}/\text{span} = 2\pi R/5 = 75.5 \simeq 75 \text{ stringers}$$

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Letting the stringers carry the entire load, the force per stringer is:

$$P_{st} = P/N = p \pi R^2 \times 5 / 2 \pi R = 5\ pR/2$$

and the required cross sectional area for each stringer is:

$$A_s = P_{st}/\sigma = \text{Force per stringer/stress}$$

This shall be evaluated in a subsequent section which combines axial and bending stress to arrive at total stress.

3.4.2 Distributed Normal Load

The stringers support the long edges of the 5 x 9 inch panels. Therefore, the load distribution along a stringer due to a single panel must be determined.
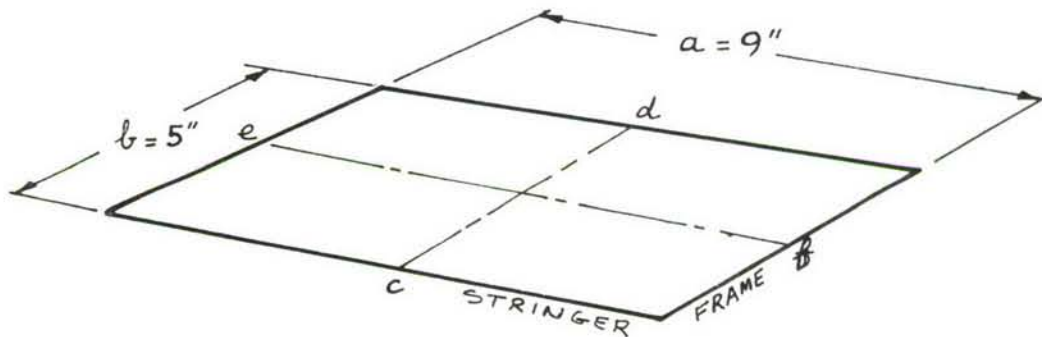
Consider a one inch wide strip from c to d as shown above. Taken as a uniformly loaded beam, the shear at c is:

$$V_c = P/2 = W_b b/2$$

where W_b = the distributed load in lb/in.

The shear at any point along the long edge may be expressed as:

$$V_x = (1/2) W_b b \, \text{Sin} \, (\pi x/a)$$

based on the assumption of a sinusoidal load distribution (Reference 9).

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Similarly, shear along the short edge may be expressed as:

$$V_y = (1/2) W_a a \sin(\pi y/b)$$

where W_a = distributed load in lb/in.

Now we have two requirements: The total reaction force must equal the applied pressure load;

$$W_b b \int_0^a \sin \pi x/a \, dx + W_a a \int_0^b \sin(\pi y/b) \, dy = pab$$

and the maximum deflections at the center must be equal

$$W_b b^4/38bEI = W_a a^4/38bEI$$

or:

$$W_a/W_b = (b/a)^4$$

Solving the above equations:

$$W_b = p\pi/\left[2(1+(b/a)^4\right]$$

$$W_a = p\pi/\left[2(1+(a/b)^4\right]$$

The total load on one long edge due to a single panel is:

$$P_s = 1/2 \, W_b b \int_0^a \sin(\pi x/a) \, dx$$
$$= pab/2.187$$

and along the short edge:

$$P_f = W_a ab/\pi = pab/23$$

For the total pressure, static plus aerodynamic (12 psi), the distributed normal load on a stringer due to one 5 x 9 inch panel is:

$$q_s = 1/2 \, W_b b = pb\pi/4 \, (1+(5/9)^4) = 12(5)\pi/4 \, (1+(5/9)^4)$$

$$= 43 \text{ lb/in at the center of the stringer.}$$

The bending moment in the stringer under sinusoidal loading is:

$$M_{max} = .0645 \, q_s a^2 = .0645(43)(9)^2$$

$$= 224 \text{ in lb}$$

and:

M_{max} = 448 in lb due to two adjacent panels.

3.4.3 Combined Stress

The combined stress in a stringer is:

$$\sigma = p/A + M/Z$$

If we design to ultimate stress, the combination of axial and bending stress must sum to σ_u. Since many stringer designs could possess the required properties, curves have been constructed relating pressure, area, and section modulus. These are shown in Figure 11. Any stringer with properties which place it on or above the curve is adequate. We will first select a member which has minimum requirements. To limit deflection, Z should be large. Minimum thickness limits this, however, as the stringer should be at least as thick as the skin being attached to it. With this as a limit, several trials were made until the hat section shown in Figure 12a was chosen. Point A on Figure 11 indicates that the section is a minimum design with:

Material = 7075-T6

A   = .067

Z   = .0082

I   = .0021

Deflection of this beam would be:

$$w = qa^4/456EI = (2 \times 43)(9)^4/456\,(10)^7\,(.0021) = 0.059 \text{ in}$$

This deflection is not acceptable. It approaches the magnitude of skin deflection relative to the stringers and would therefore tend to increase the panel loads. The design is also unacceptable because:

FIGURE II. MINIMUM REQUIRED STRINGER SECTION PROPERTIES
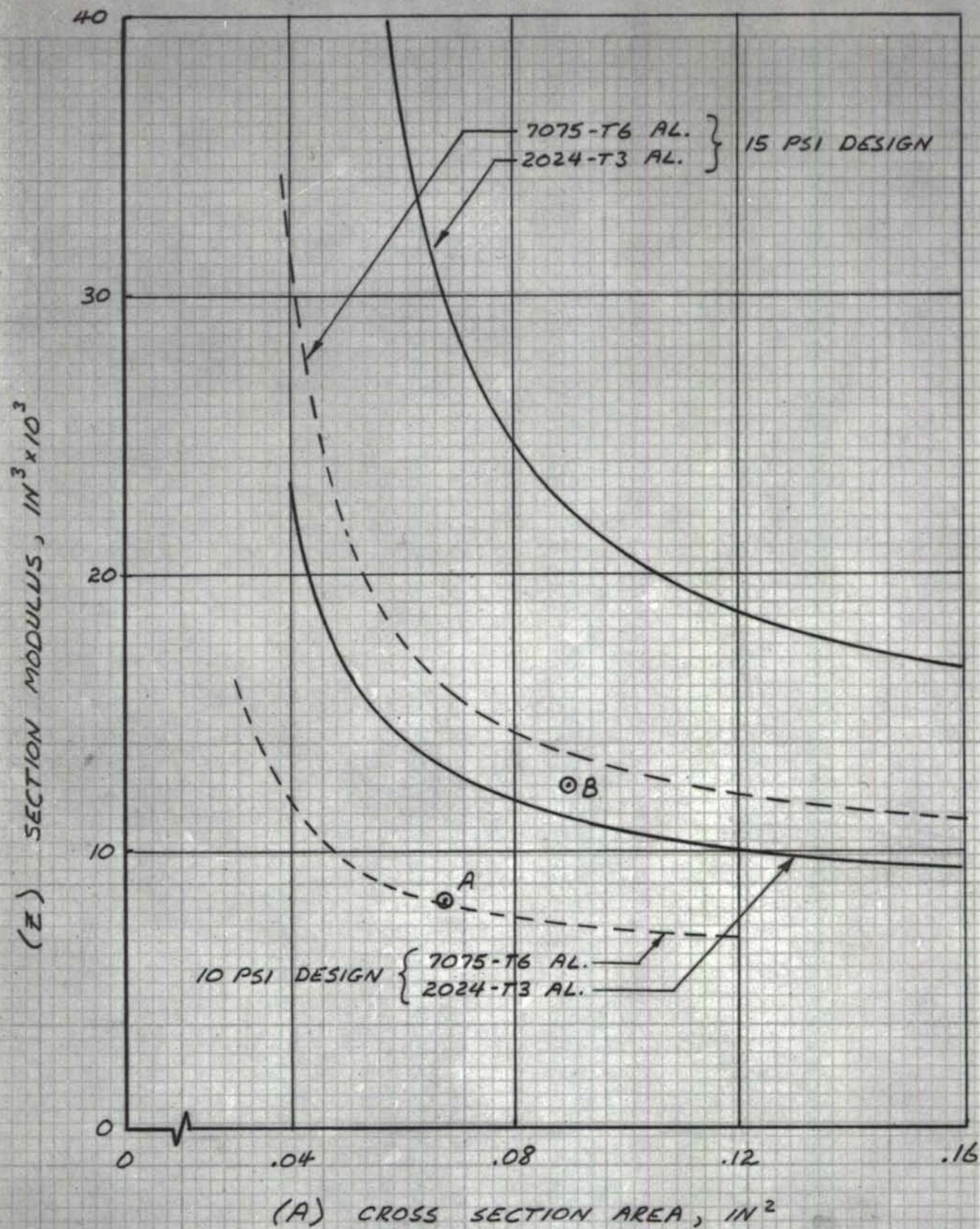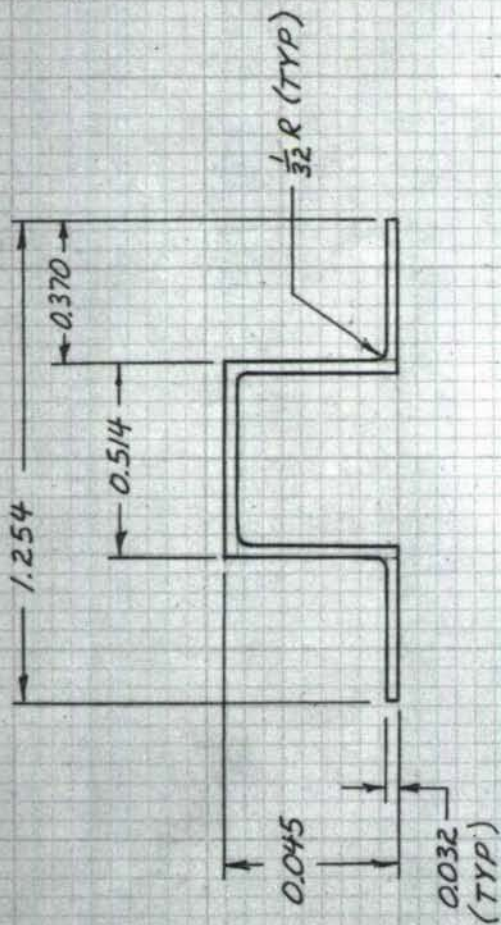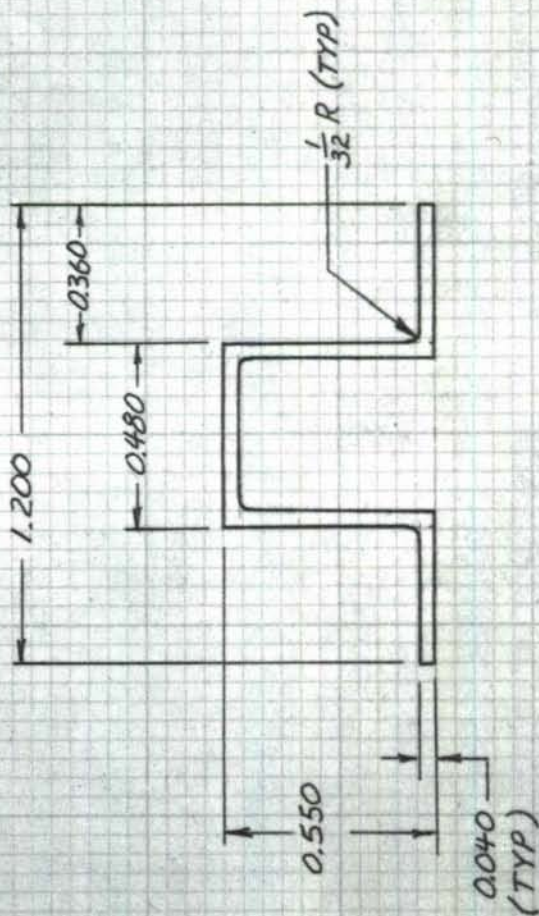
(a)

AREA = 0.067 IN.
INERTIA = 0.0021 IN.^{4}
$\overline{Z}$ = I/c = 0.0082 IN.^{3}

(b)

AREA = 0.089 IN
INERTIA = 0.0039 IN^4
$\overline{Z}$ = I/c = 0.0125 IN^3

FIGURE 12. HAT SECTIONS - INTERMEDIATE DESIGN

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.1. Height of hat would limit design to 1/16 rivets to attach corner supports. Four rivets, 2 per bracket, would not have shear strength to support stringer relative to frames.

2. Section too flimsy. Low torsional stiffness.

3. Rivet installation would be critical, edge distance minimum, chance of bad installation inside hat.

.4. If extrusions were chosen, the thin wall would be difficult to extrude and warping due to heat treatment would be a problem.

One solution is to go to a different aluminum alloy for the stringers. For 2024-T3 extrusion, ultimate stress is 57,000 psi. This lower stress requires more area and/or a larger section modulus (Z). Therefore, dimensions would increase, larger rivets would be possible, and deflection would decrease. Figure 12b shows a possible configuration for a hat section. This section's area and modulus are indicated as point B in Figure 11. The deflection for this section is:

$$w = qa^4/456EI = (2 \times 43)(9)^4/456(10)^7(.00389) = 0.032 \text{ in.}$$

This deflection is marginal. Therefore, another method will be tried.

3.4.4 Deflection Limited Design

The above analysis attempted to design the stringers to ultimate stress under the imposed loads. This approach is possible; but, as indicated above, it results in a marginal design with difficult rivet installation and relatively large deflection.

The most realistic mode of failure due to acoustic loads is failure of the skin, rather than the stringers. To assure skin failure the stringers

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are designed to an arbitrary deflection limit rather than to ultimate stress. This, of course, relieves all problems associated with the small section.

Proceeding with this method, sections are selected which limit deflection to approximately 0.010 inch and allow use of 3/32 or larger rivets. The moment of inertia required for the 10 psi design is:

$$I = qa^4/456Ew = (2 \times 43)(9)^4/456(10)^7(.010) = .0124 \text{ in}^4$$

and for the 15 psig design:

$$I = (2 \times 68)(9)^4/456(10)^7(.010) = .0196 \text{ in}^4$$

Sections have been selected which meet or approach these requirements. They are shown in Figure 13 and their properties are listed in the following table:

Figure No.	Type Section	Area in^2	Moment of Inertia in^4	Z in^3	Deflection Max. In.
13c	hat	0.118	0.0096	0.021	.013
13d	hat	0.184	0.028	0.049	.007
13a	J	0.110	0.0098	0.021	.007
13b	J	0.175	0.028	0.046	.007

3.4 EFFECTIVE PANEL WIDTH

Now that the stringer cross sections have been selected, it is possible to verify the skin thicknesses required by going through the analysis with a panel width based on the stringer rivet spacing.

Consider the 10 psi design with J section stringers. The stringer is riveted to the panel with a double row of rivets spaced .50 inches apart. The effective width of the panel is now 4.50 inches, the span between rows of rivets. Using this width in the analysis outlined in paragraph 3.3.3 yields a skin thickness requirement of .0265 inches. The .032 sheet is still the

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FIGURE 13. SELECTED STRINGER CROSS-SECTIONS.
a) J-SECTION, 10 PSI DESIGN
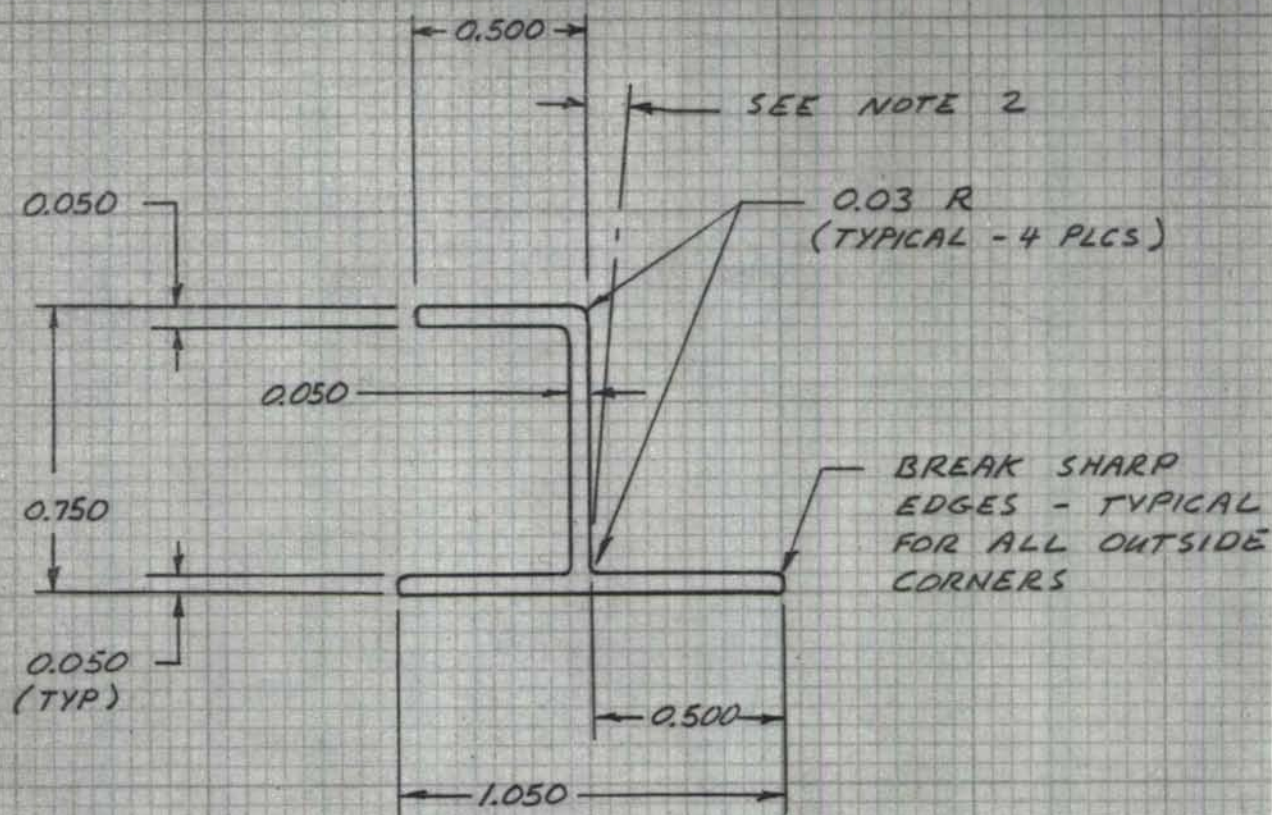
NOTES: 1. MAT'L — 2024-T4 OR EQUIVALENT
2. ANGULARITY — ±2°
3. THIS CROSS-SECTION REQUIRES AN EXTRUSION DIE
4. SCALE, 2:1
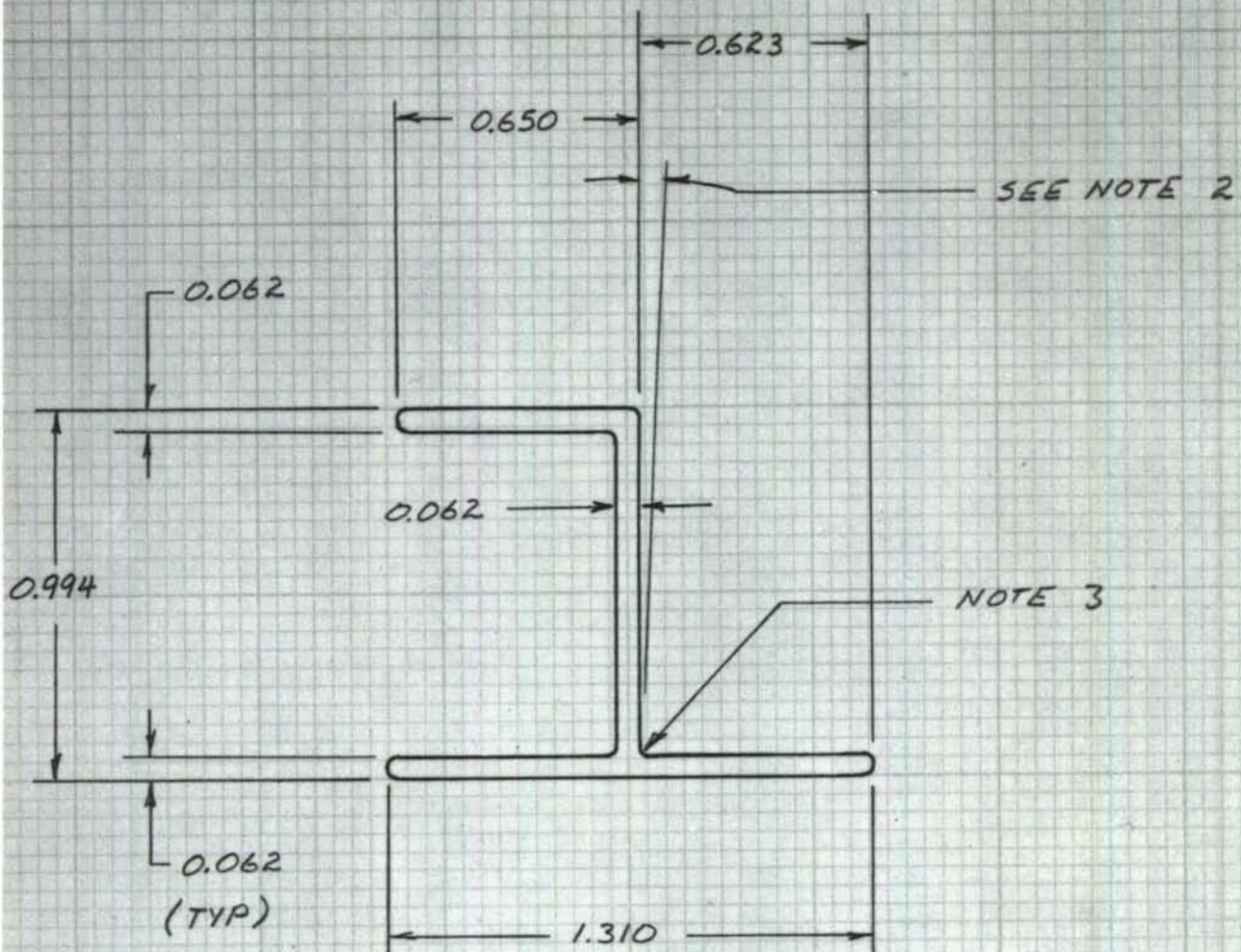
NOTES: 1. MAT'L , 2024-T4 OR EQUIVALENT

2. ANGULARITY , ± 2°

3. ALL CORNER RADII 0.06

4. THIS CROSS SECTION CORRESPONDS TO PIONEER ALUMINUM EXTRUSION NO. PA 13499

5. DWG. SCALE 2:1

FIGURE 13 - CONTINUED.

b) J- SECTION , 15 PSI DESIGN
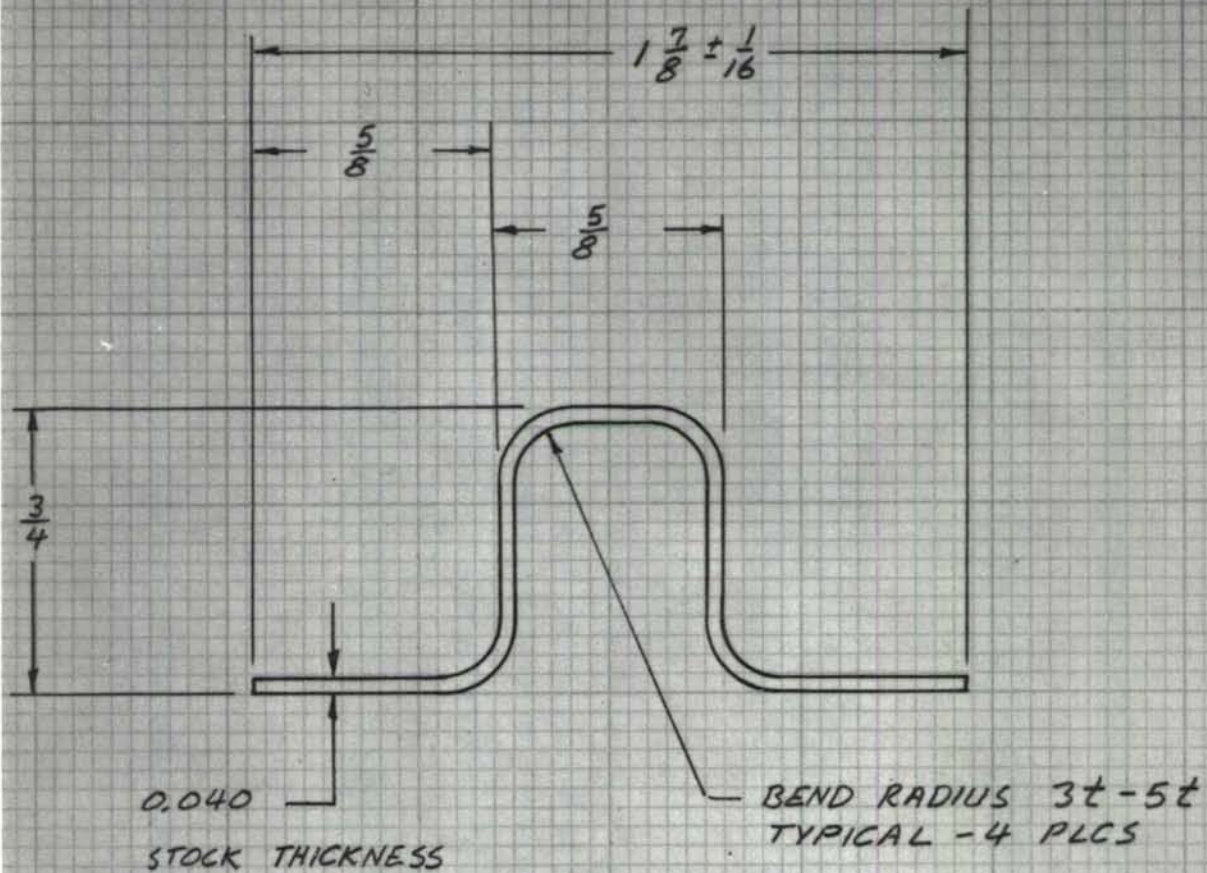
NOTES:
1. MAT'L, 2024-T3 SHEET
2. TOLERANCE ON FRACTIONAL DIMENSIONS $\pm\frac{1}{32}$
3. DWG SCALE 2:1

FIGURE 13 - CONTINUED

c) HAT SECTION, 10 PSI DESIGN

$2\frac{1}{16} + \frac{1}{16} - \frac{1}{16}$

$\frac{5}{8}$

$\frac{13}{16}$

1

0.050
STOCK

BEND RADIUS $3t - 5t$
TYP - 4 PLACES

NOTES : 1. MAT'L, 2024-T3 SHEET
   2. TOLERANCE ON FRACTIONAL
      DIMENSIONS ± $\frac{1}{32}$
   3. DWG. SCALE, 2:1

FIGURE 13 - CONCLUDED

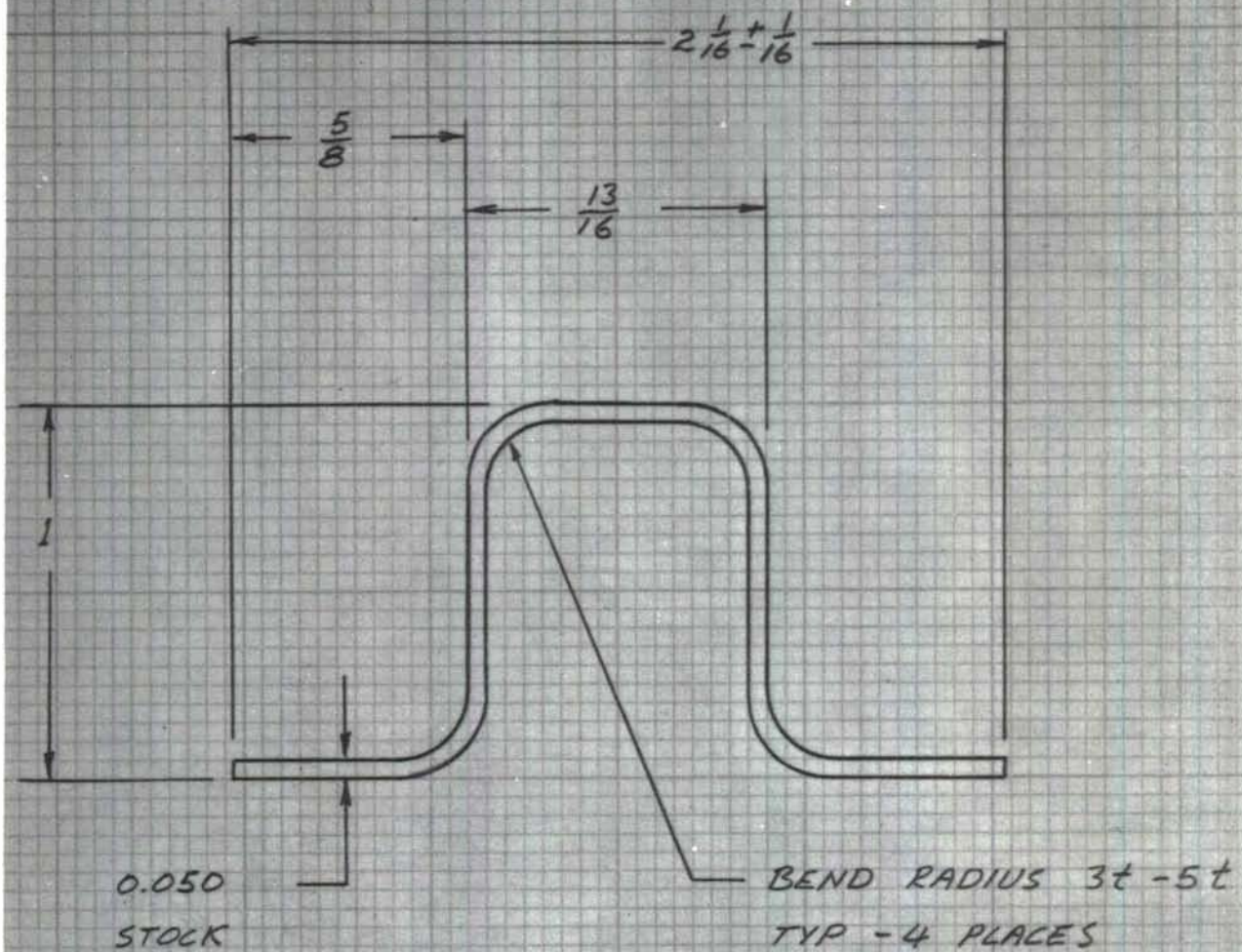d) HAT SECTION, 15 PSI DESIGN

nearest commercial size. The 15 psi design with J section stringers has an effective panel width of 4.2 inches, resulting in a skin thickness requirement of .034 in. Again, the .040 in sheet previously selected is the nearest commercial size.

The hat section stringers have a wide rivet spacing but they cannot be considered rigid at the rivet line. They are made of sheet and deform under load with the skin. The effective panel width is reasonably assumed to be the same as that of the J section panel. Thus all designs stand with .032 in sheet for the 10 psi design and .040 in sheet for the 15 psi design.

3.5 DESIGN OF FRAMES

On the basis of tension loads due to a pressure loading on the cylinder, the minimum frame area may be computed assuming ultimate stress for 2024-T3 and substituting in the hoop stress formula. Taking the total hoop load out through the frames gives:

$$\sigma = \text{Load}/\text{Area} = p(\text{width})2R/2A$$

or
$$A = (12 \times 9 \times 60)/(72000 \times 2)$$
$$A = 0.045 \text{ in}^2$$

for the 15 psi design:
$$A = 0.056 \text{ in}^2$$

These areas are conservative since the aerodynamic pressure which is included does not act around the entire circumference of the shell. The required minimum areas are approximately the same as the stringer areas; therefore, it may be concluded that hoop loads alone do not design the frames.

The selection of frames is thus arbitrary and sizes are selected to be compatible with the skin and stringers. A thickness of approximately twice the skin thickness is selected. Height is taken to be about 2 1/2 times the stringer height. Figure 14 shows the frame cross section selected. Dimensions were taken from the Army-Navy Aeronautical Design Standard AND10138.

3.6 RIVET SELECTION

Rivet spacing is one inch or less to assure even load transfer from sheet to stringers. Rivet diameters are selected to insure adequate margin of safety. AD type rivets with 30,000 psi ultimate stress are used throughout.
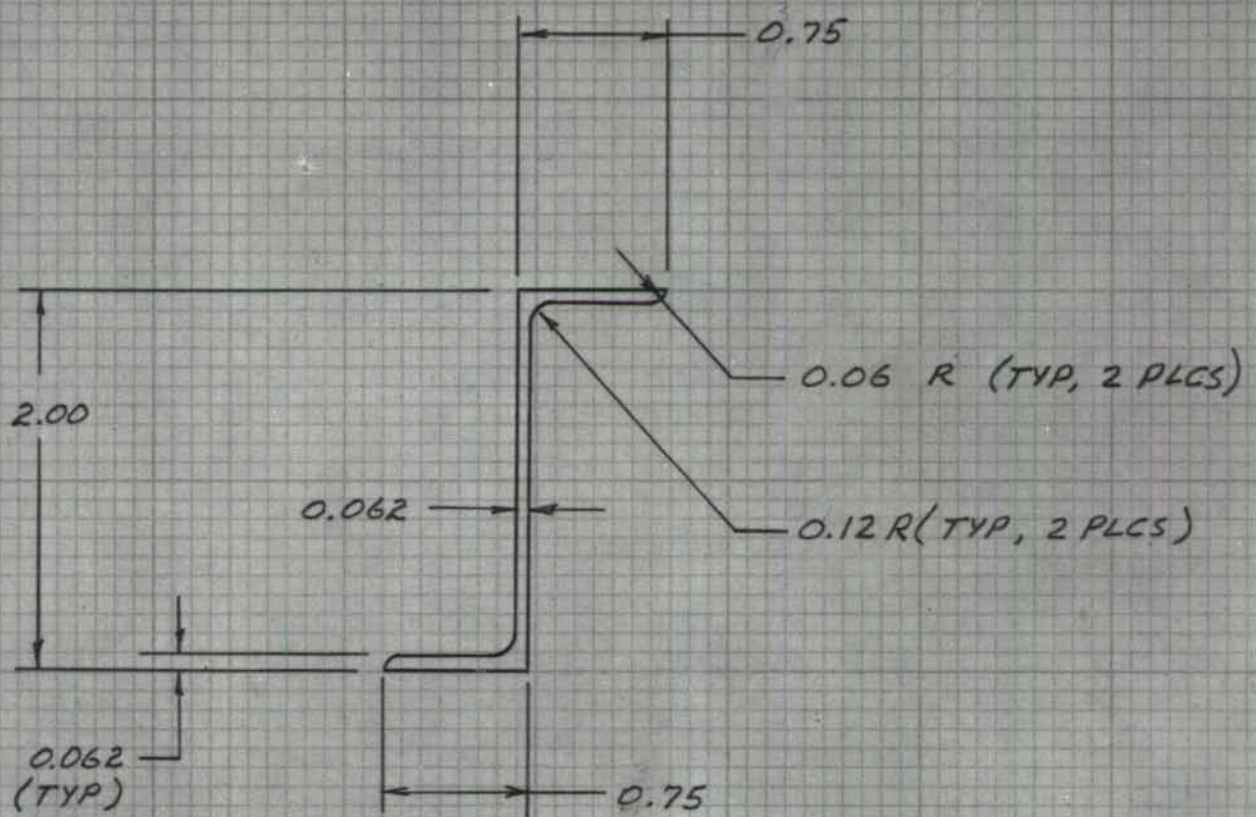
3.6.1 Skin To Stringers and Frames

The skin to stringer attachment is made with a row of rivets through each flange of the stringer. The following discussion applies to one row only.

Recall that the maximum normal stringer load was 43 lb/in. at the center of the span due to one skin panel only. For a nominal one inch rivet spacing the rivet strength must be greater than 43 lbs. (or 68 lbs for the 15 psi design). Rivets of 1/16 diameter have an 80 lb. capacity which would be more adequate. 3/32 diameter rivets, however, are the minimum recommended size for aircraft construction, and are used for all specimens.

The previous analysis of the normal loads between the skin and the frames indicated a maximum load at center of the five inch span of:

$$q_f = 1/2 \ W_a a = pa\pi / \left[4(1 + (a/b)^4) \right]$$

$$= 12(9)\pi / 4(1 + (9/5)^4)$$

$$= 7.36 \ \text{lb/in}$$

0.75

0.06 R (TYP, 2 PLCS)

2.00

0.062

0.12 R (TYP, 2 PLCS)

0.062
(TYP)

0.75

NOTES : 1. MAT'L, 2024-T4 OR EQUIVALENT
2. TOLERANCE ON DIMENSIONS, ±0.03 & ±0.010
3. THIS CROSS SECTION IS EQUIVALENT
    TO AND10138-2001
4. DWG SCALE, FULL SIZE

FIGURE 14. SELECTED FRAME CROSS-SECTIONS.
    a) FRAME CROSS-SECTION,
    10 PSI DESIGNS

0.88

2.50

0.078

0.06 R (TYP - 2 PLCS)

0.12 R (TYP - 2 PLCS)

0.078 (TYP)

0.88
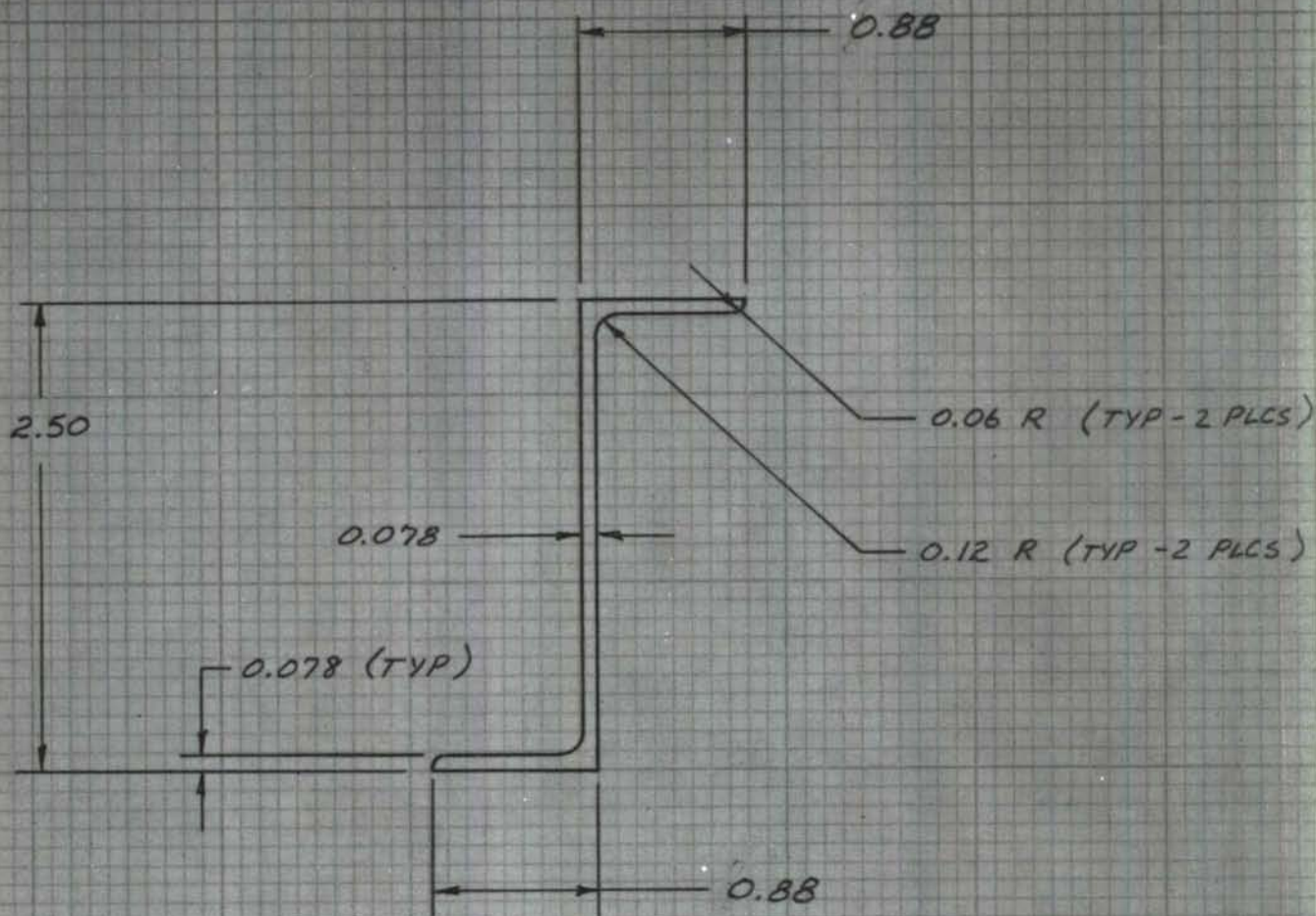
NOTES : 1. MAT'L, 2024-T4 OR EQUIVALENT

2. TOLERANCE ON DIMENSIONS, .XX ± 0.03
.XXX ± 0.010

3. THIS CROSS SECTION IS EQUIVALENT
TO AND 10138 - 2402

4. DWG SCALE , FULL SIZE

FIGURE 14 - CONCLUDED.

b) FRAME CROSS - SECTION,
15. PSI DESIGNS

due to one panel. Two panels load it to 14.8 lb/inch, well within the capacity of 3/32 rivets. The 15 psi design loads the frame to 23.4 lb/in; also within the capacity of 3/32 rivets.

3.6.2 Stringers to Frames

The load transferred from a stringer to a frame for the 10 psi design is:

$$P_{st} = (pab/2.187)2$$
$$= (12 \times 5 \times 9/2.187)2$$
$$= 495 \text{ lb.}$$

And for the 15 psi design:

$$P_{st} = 785 \text{ lb.}$$

The joint is held by two clips as shown in Figures 15 thru 18. The clip design is shown in Figure 19. There are thus four rivets in single shear carrying the load. Four 3/32 diameter rivets carry $4 \times 217 = 868$ lb. in shear (ultimate) which is adequate for all specimens. The margin of safety is 75% for the 10 psi design and 10% for the 15 psi design. The frame cutouts for the various configurations are shown in Figures 20 thru 23.

3.7 DESIGN SUMMARY

The frame stringer panel designs are summarized as follows. The skin thicknesses required are 0.032 and 0.040 for the 10 psi and 15 psi designs respectively. The stringer cross-sections are shown in Figure 13, frame cross-sections are shown in Figure 14, and Figures 15 thru 18 present frame stringer intersections. The basic layout of frames, stringers and rivet patterns for all specimens is shown in Figures 24 thru 27.

FRAME

STRINGER

CLIP, 4 REQ'D

MS 20470AD3 RIVET, 6 REQ'D
DRILL HOLES ON ASSEMBLY

3/8

4/16 NOMINAL, CENTER STRINGER IN CUTOUT

SCALE: FULL SIZE
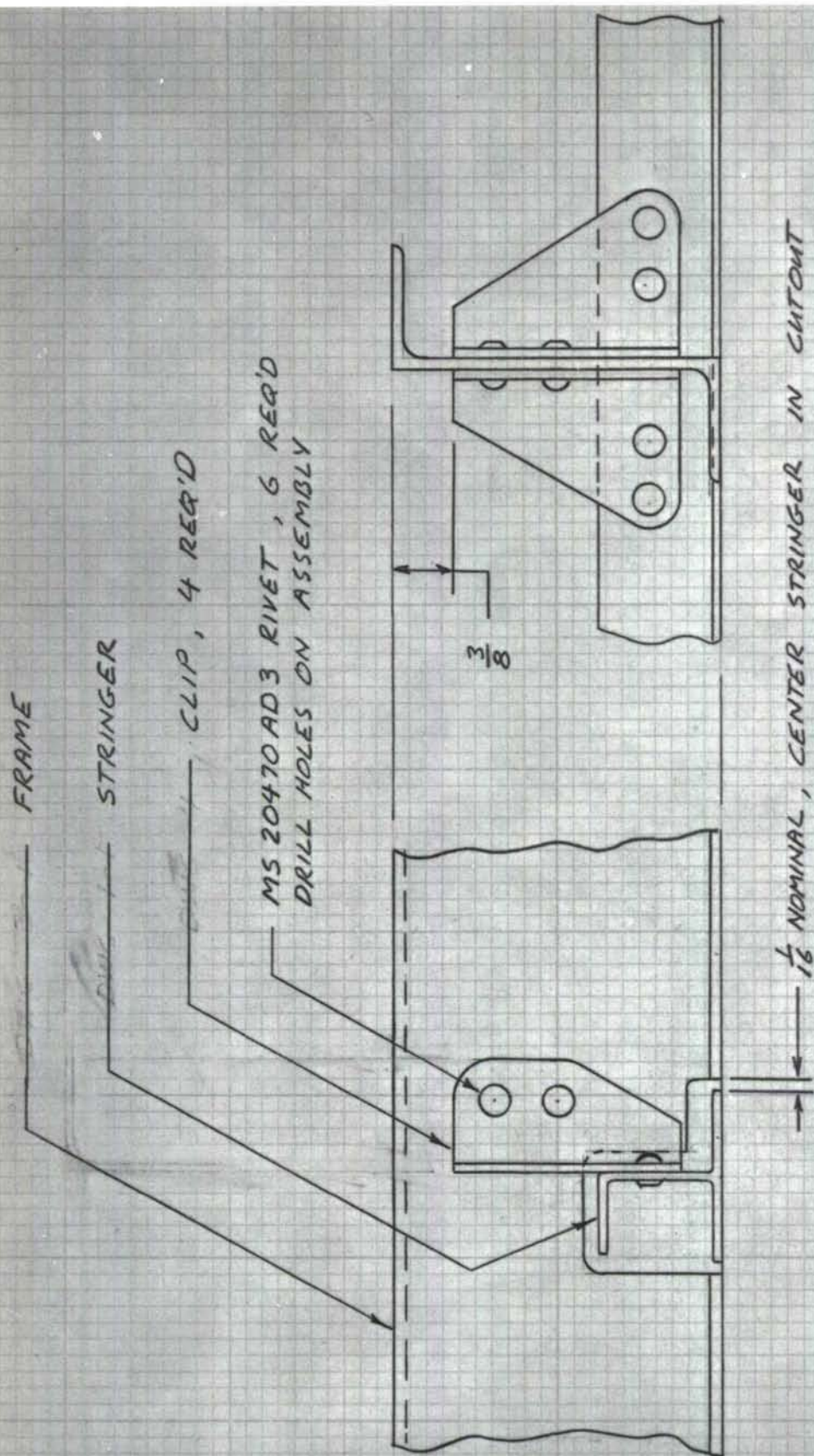
FIGURE 15. TYPICAL FRAME - J STRINGER

INTERSECTION FOR 10 PSI DESIGNS

50

FRAME

STRINGER

CLIP, 2 REQ'D

MS 20470AD3 RIVET, 6 REQ'D
DRILL HOLES ON ASSEMBLY

0.80

$\frac{1}{16}$ NOMINAL, CENTER STRINGER IN CUT-OUT

SCALE: FULL SIZE
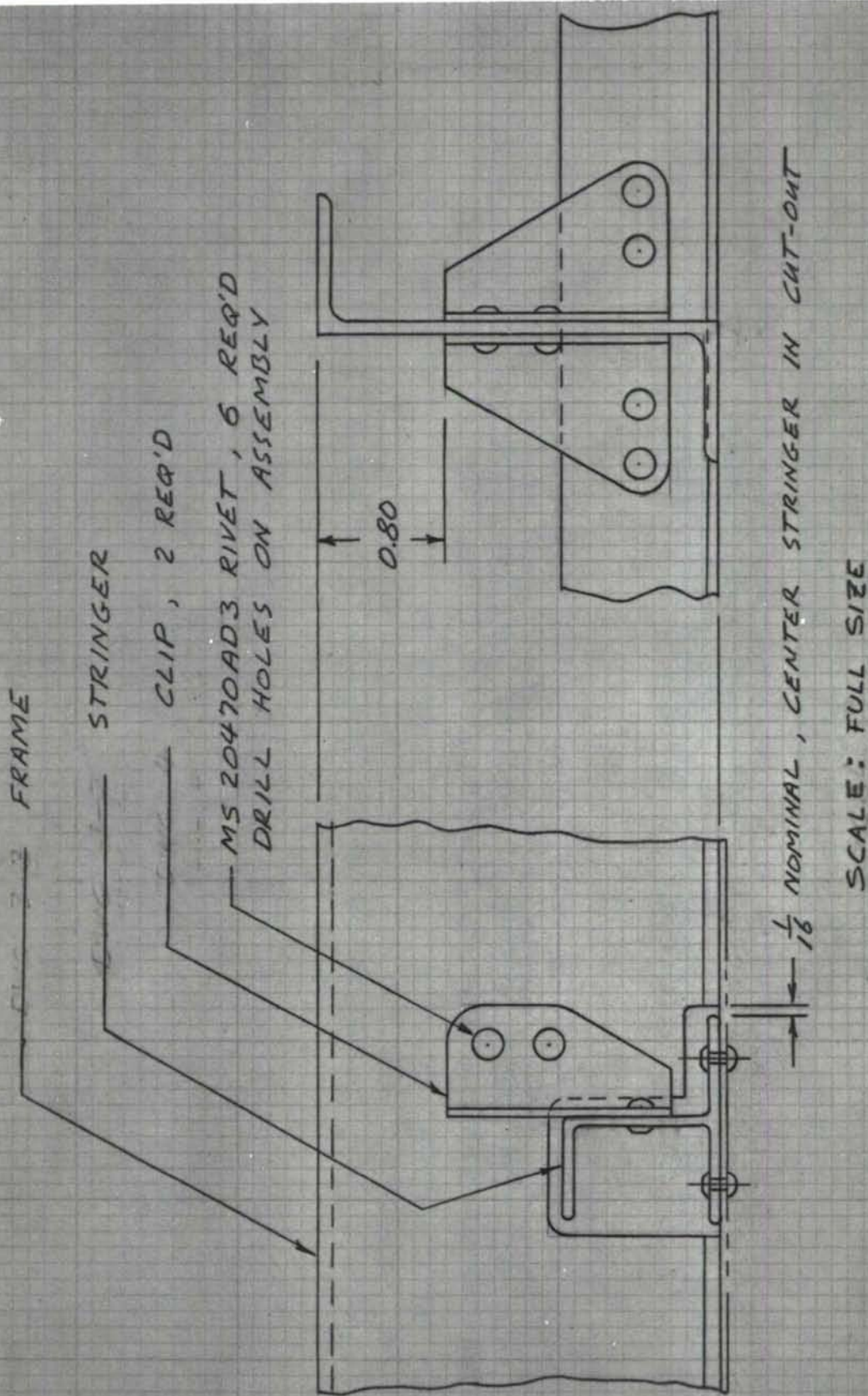
FIGURE 16. TYPICAL FRAME - J STRINGER
INTERSECTION FOR 15 PSI DESIGN

FRAME

STRINGER

CLIP

MS 20470 AD3 RIVET, 8 REQ'D
DRILL ON ASSEMBLY

0.43

1/16 NOMINAL, CENTER STRINGER IN CUTOUT

SCALE: FULL SIZE
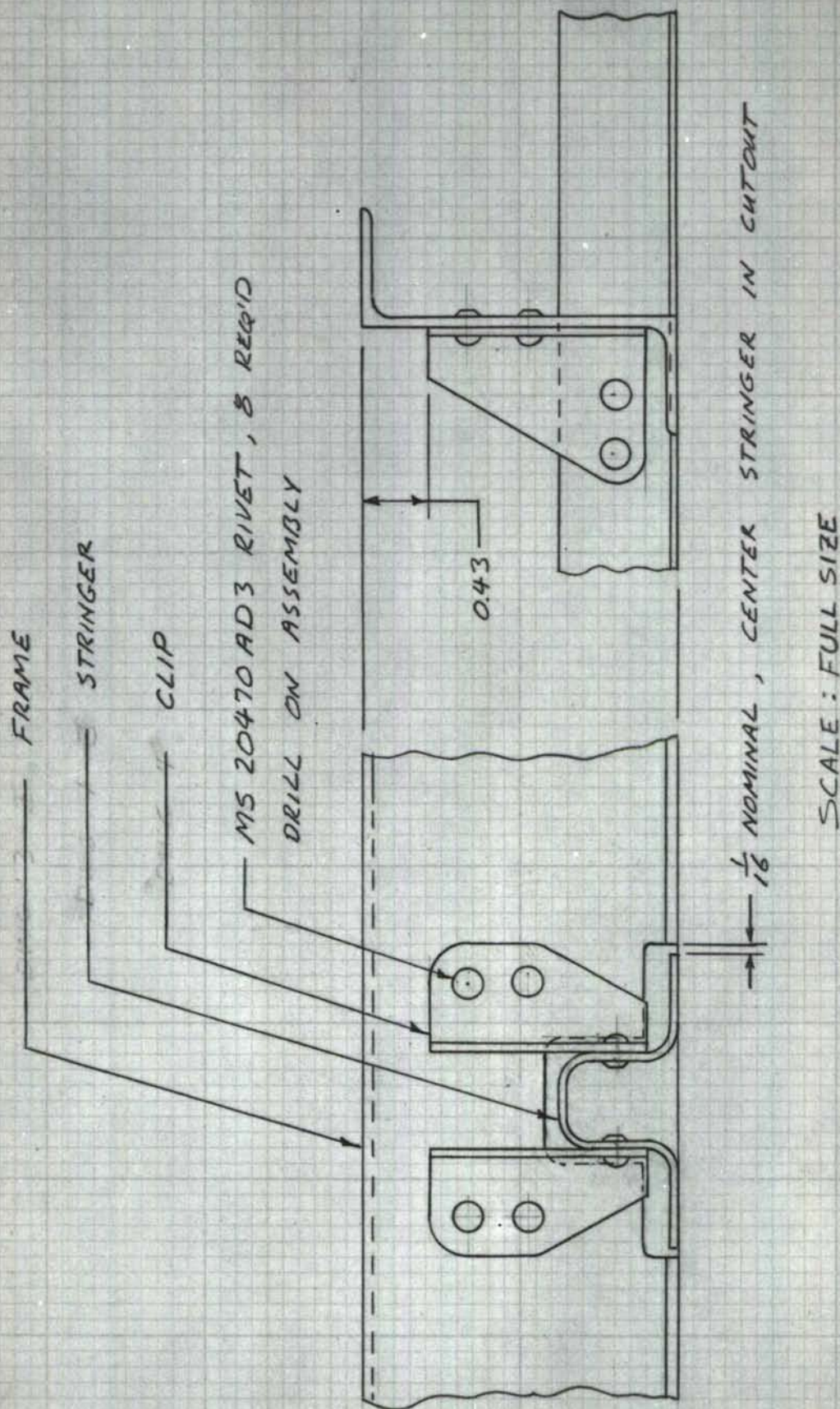
FIGURE 17. TYPICAL FRAME-HAT STRINGER
INTERSECTION FOR 10 PSI DESIGNS

52

CLIP

STRINGER

MS 20470 AD3 RIVET, 8 REQ'D, DRILL ON ASSEMBLY

FRAME

0.80

$\frac{1}{16}$ NOMINAL, CENTER STRINGER IN CUTOUT

SCALE: FULL SIZE
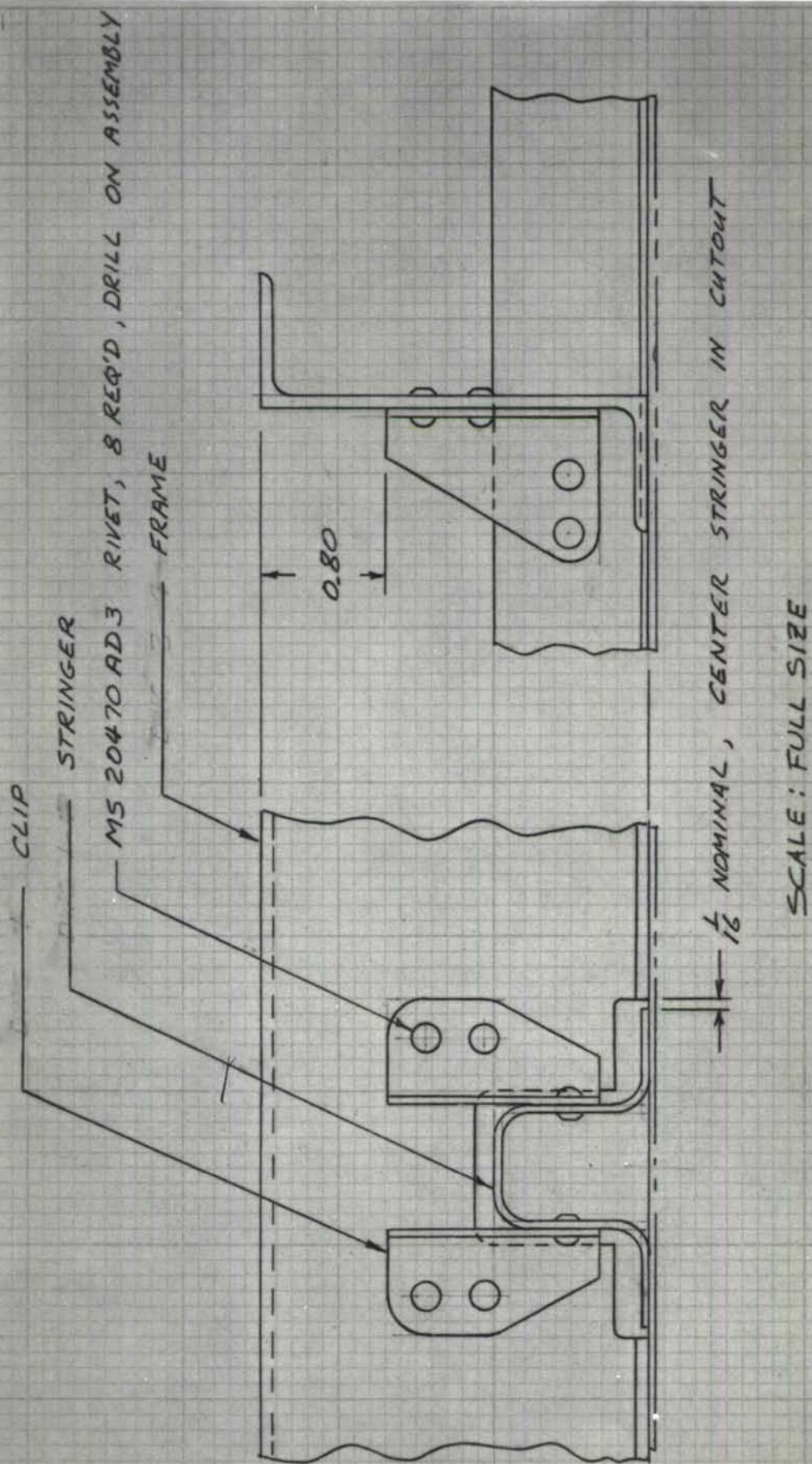
FIGURE 18. TYPICAL FRAME - HAT STRINGER
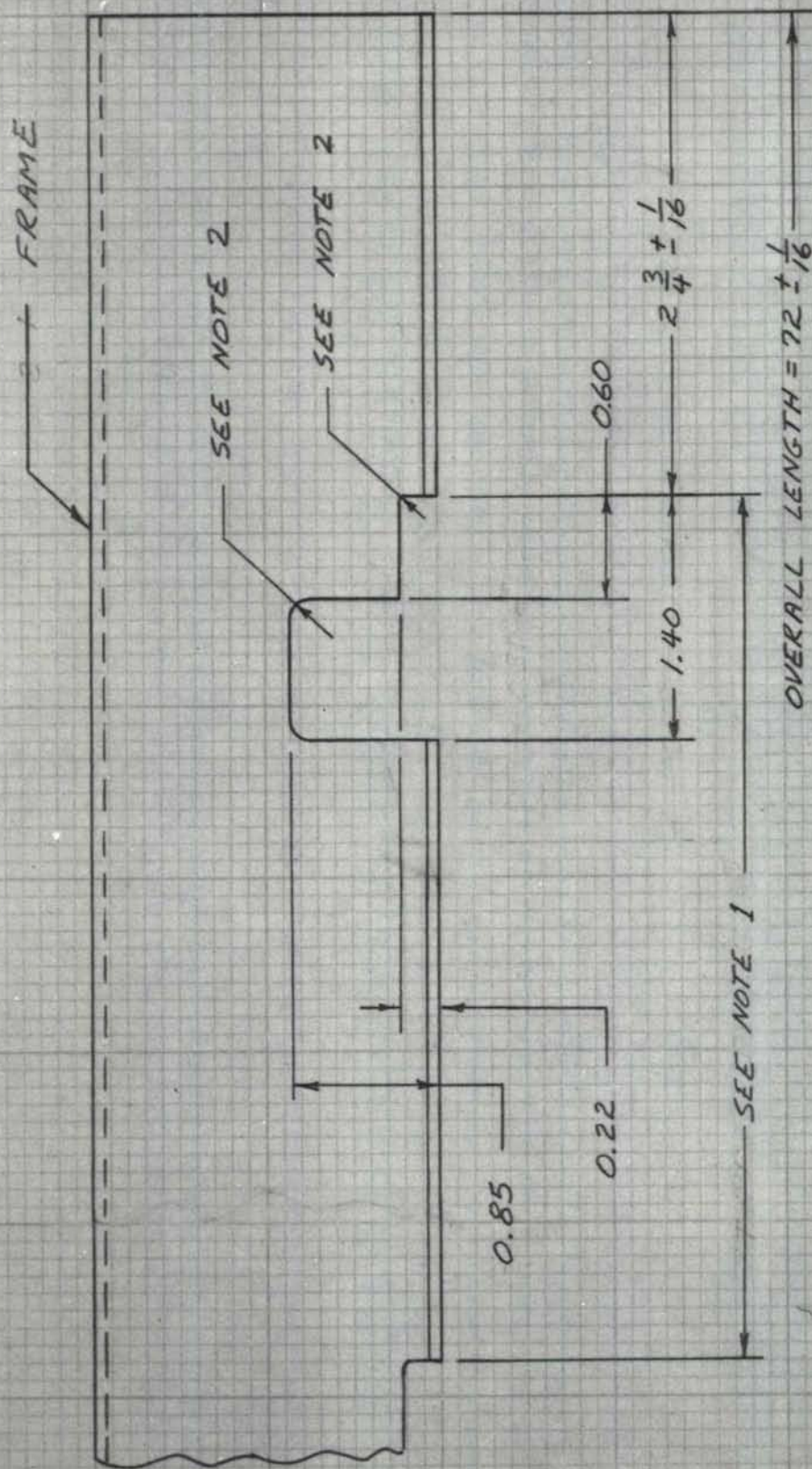
INTERSECTION FOR 15 PSI DESIGN

$R = 3t - 5t$

$\frac{11}{16}$

1

$\frac{1}{4} R$ (TYP)

$\frac{1}{4}$

$\frac{1}{4}$

$\frac{5}{16}$

$1\frac{3}{8}$

$\frac{3}{8}$

$\frac{3}{16}$

#40 PILOT HOLES
(TYP 4 PLCS)

$\frac{3}{16} R$

$\frac{3}{8}$

NOTES : 1. MAT'L : 0.040 THICK 2024-T3 AL. SHT
2. TOLERANCE, FRACTIONS $\pm \frac{1}{32}$

FIGURE 19. CLIP

54

NOTES: 1. THERE MUST BE 13 EQUAL SPACES OVER A 65 INCH LENGTH WITH CUT-OUTS ON $5 \pm \frac{1}{32}$ IN. CENTERS.

2. INTERNAL RADII WILL DEPEND ON MACHINING METHOD SELECTED, BUT SHALL BE WITHIN $\frac{1}{64}$ TO $\frac{1}{8}$ R

3. DWG. SCALE, FULL SIZE

4. THIS FIG. APPLICABLE TO STRAIGHT & CURVED FRAMES

FIGURE 20. MACHINED FRAME FOR 10 PSI DESIGNS WITH J-SECTION STRINGERS

NOTES : 1. 13 CUT-OUTS EQUALLY SPACED OVER A 65 INCH LENGTH,
TOLERANCE $\pm \frac{1}{32}$ BETWEEN CENTERS.

2. SCALE - FULL SIZE

3. THIS FIGURE APPLIES TO BOTH STRAIGHT AND
CURVED BEAMS.

FIGURE 21.- MACHINED FRAME FOR 10 PSI DESIGNS
WITH HAT SECTION STRINGERS

56

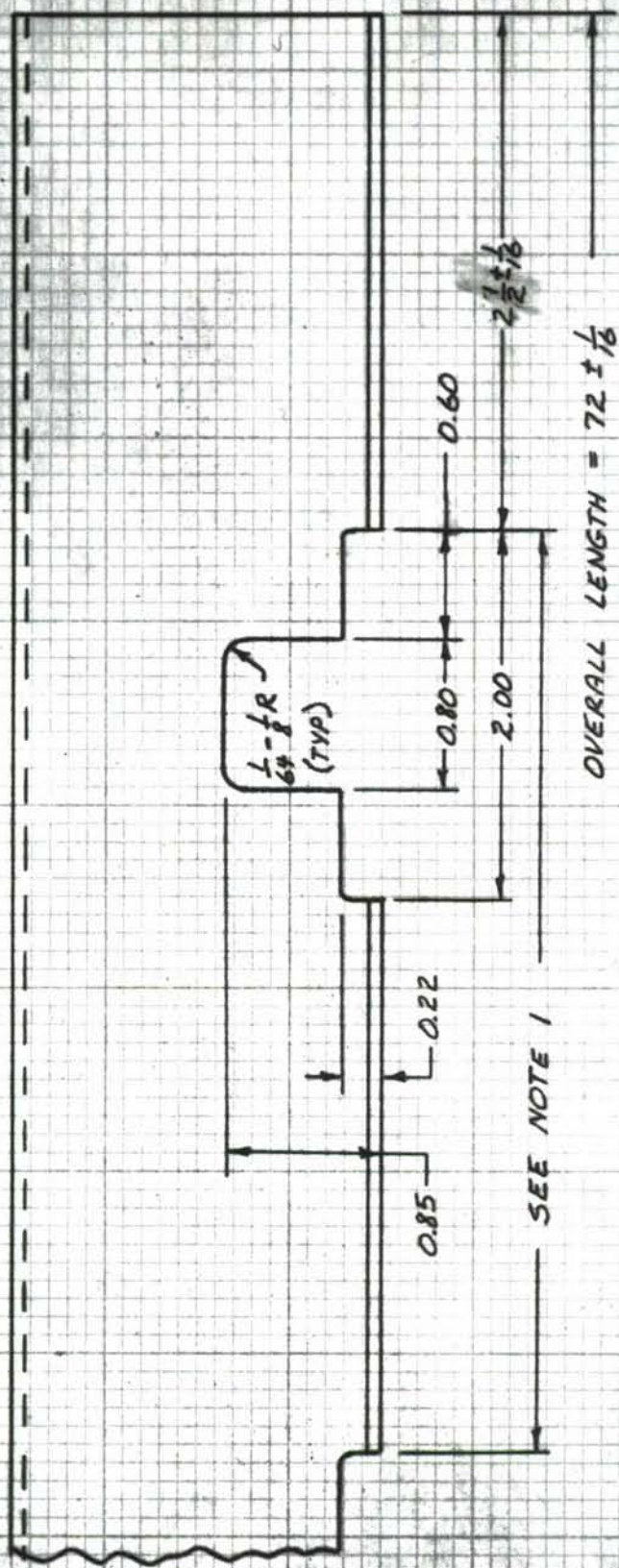NOTES: 1. THERE MUST BE 13 EQUAL SPACES OVER A 65 INCH LENGTH WITH CUT-OUTS ON 5 ± $\frac{1}{32}$ IN. CENTERS.

2. INTERNAL RADII WILL DEPEND ON MACHINING METHOD SELECTED, BUT SHALL BE WITHIN 6 $\frac{1}{4}$ TO $\frac{1}{8}$ R

3. SCALE, FULL SIZE

4. THIS FIG. APPLICABLE TO STRAIGHT & CURVED FRAMES

FIGURE 22. MACHINED FRAME FOR 15 PSI DESIGNS WITH "J" SECTION STRINGERS

$\frac{1}{64} - \frac{1}{8} R$ (TYP)
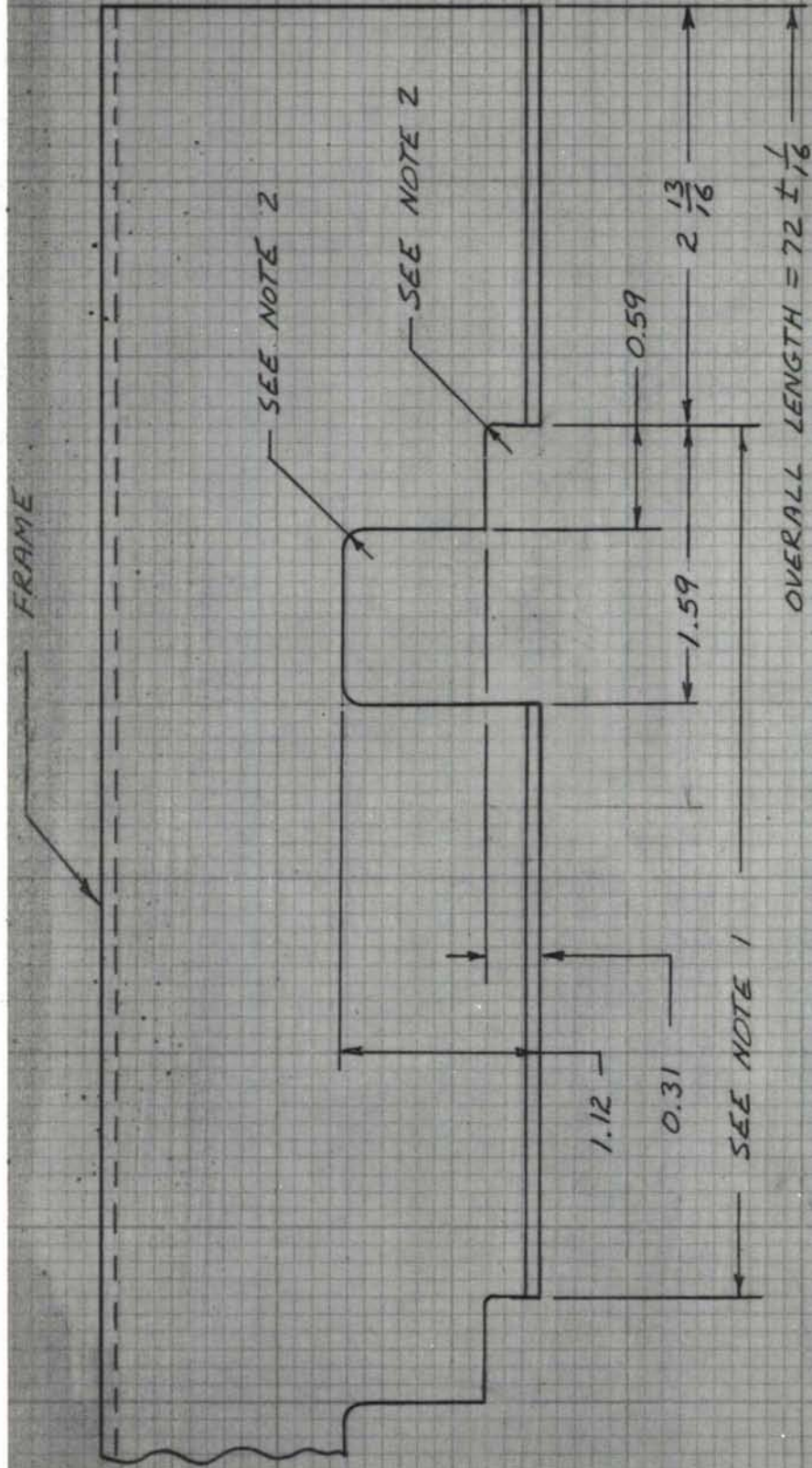
0.59

2.41

1.00

2.19

0.31

1.12

SEE NOTE 1

OVERALL LENGTH = 72 ± $\frac{1}{16}$

NOTES : 1. 13 CUT-OUTS EQUALLY SPACED OVER A 65 INCH LENGTH, TOLERANCE ± $\frac{1}{32}$ BETWEEN CENTERS.

2. SCALE : FULL SIZE

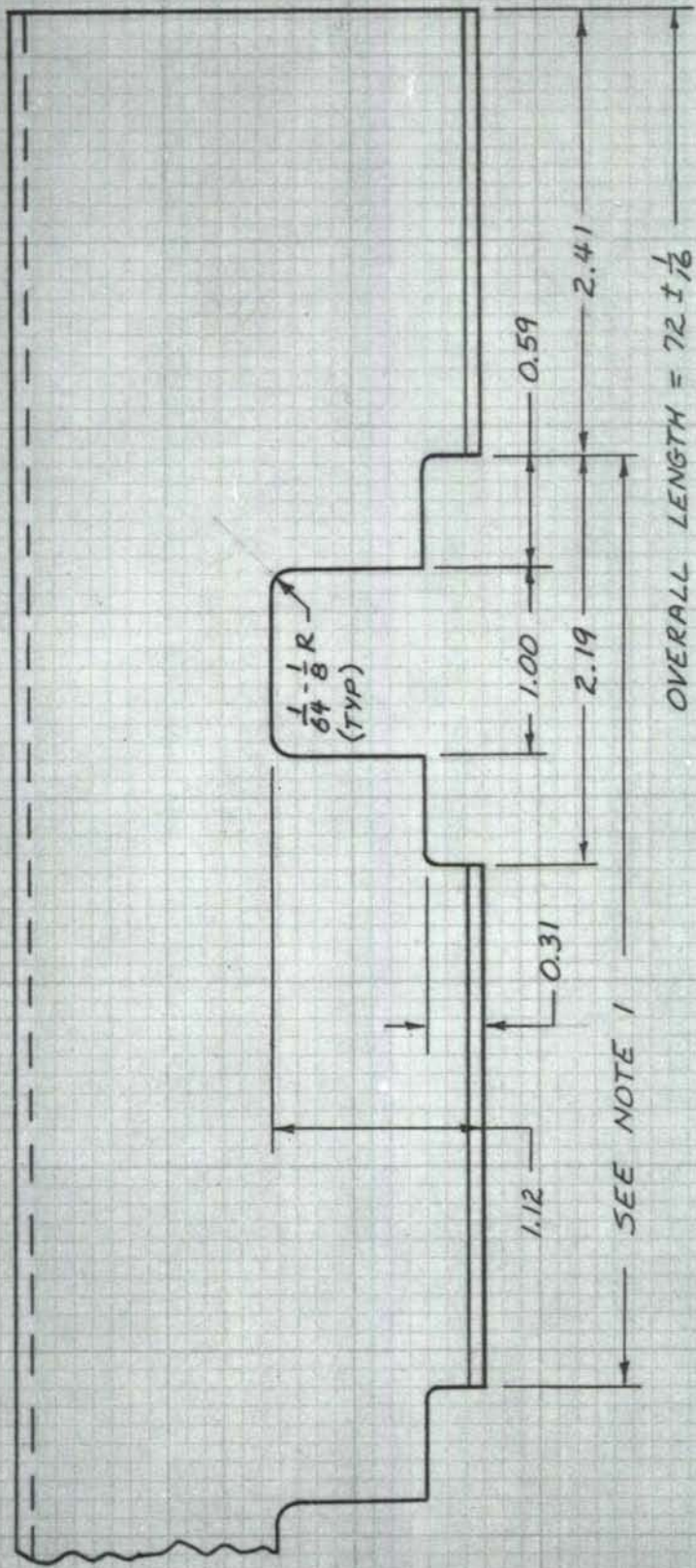3. THIS FIGURE APPLIES TO BOTH STRAIGHT AND CURVED FRAMES

FIGURE 23. MACHINED FRAME FOR 15 PSI DESIGNS WITH HAT SECTION STRINGERS

13 EQUAL SPACES (REF) STRINGERS LOCATED BY FRAME CUTOUTS

LOCATED BY FRAME CUTOUT

$3\frac{1}{2}$ NOMINAL

5 EQUAL SPACES AT 9 IN. CENTERS

$1\frac{1}{2}$

48

45

72

TOLERANCE:
FRACTIONS $\pm\frac{1}{16}$

VIEW (a) FOR 10 PSI DESIGN (DESIGN NO. 1)
VIEW (b) FOR 15 PSI DESIGN (DESIGN NO. 2)
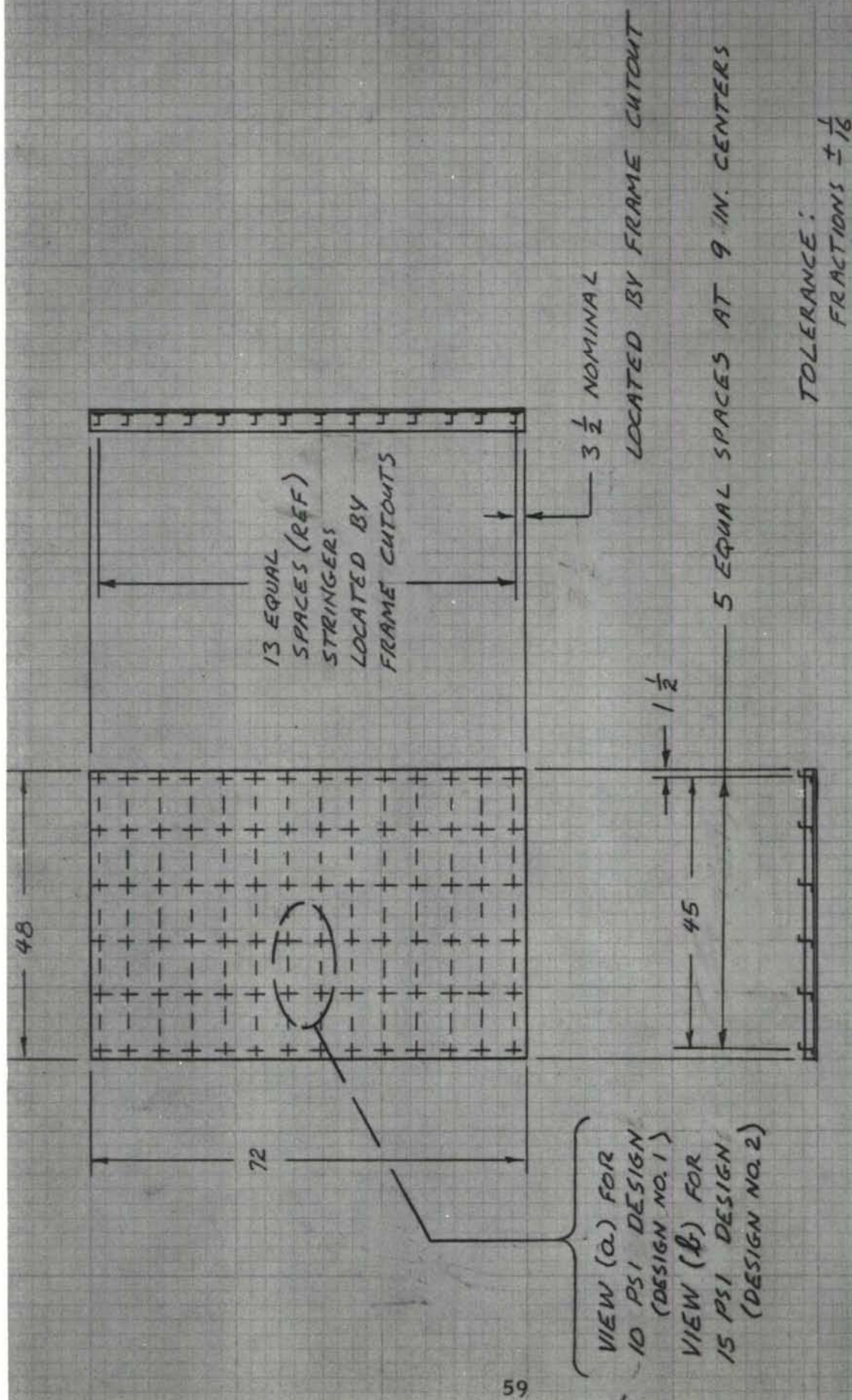
FIGURE 24. FLAT PANEL — J STRINGERS, DESIGNS 1 AND 2.
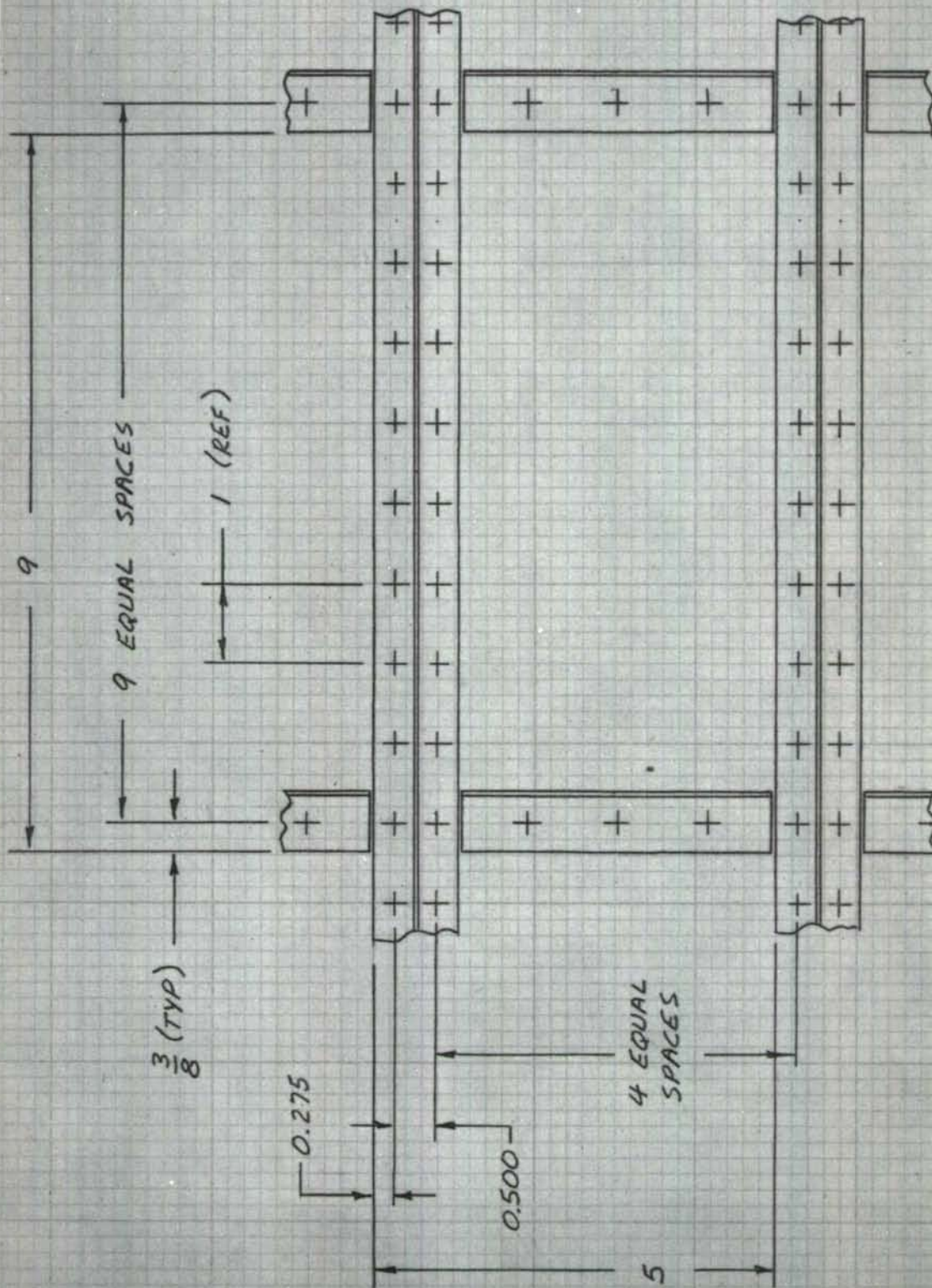
FIGURE 24 — CONTINUED.
a) 10 P.S.I. DESIGN
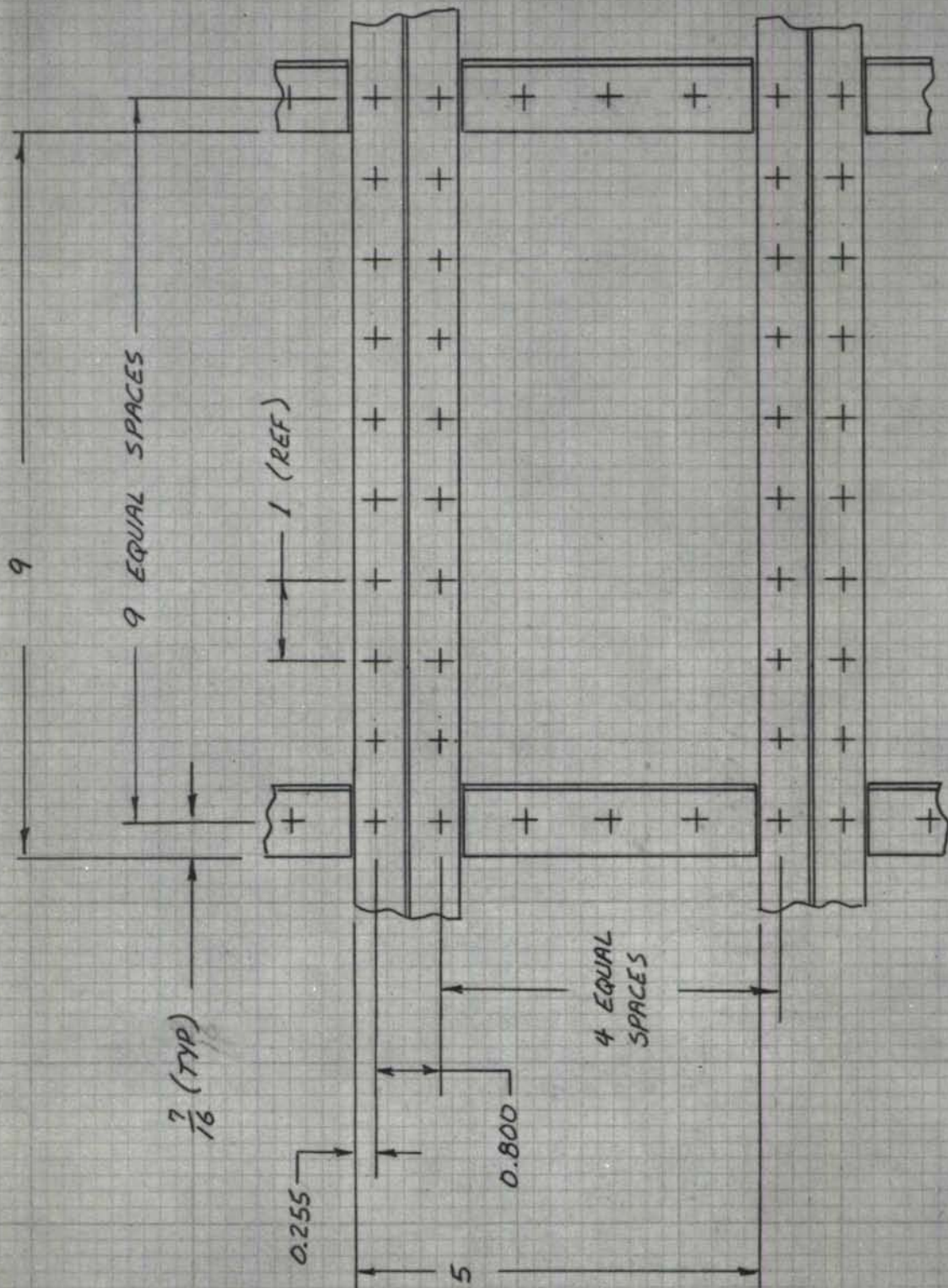
FIGURE 24 --- CONCLUDED
b) 15 P.S.I. DESIGN
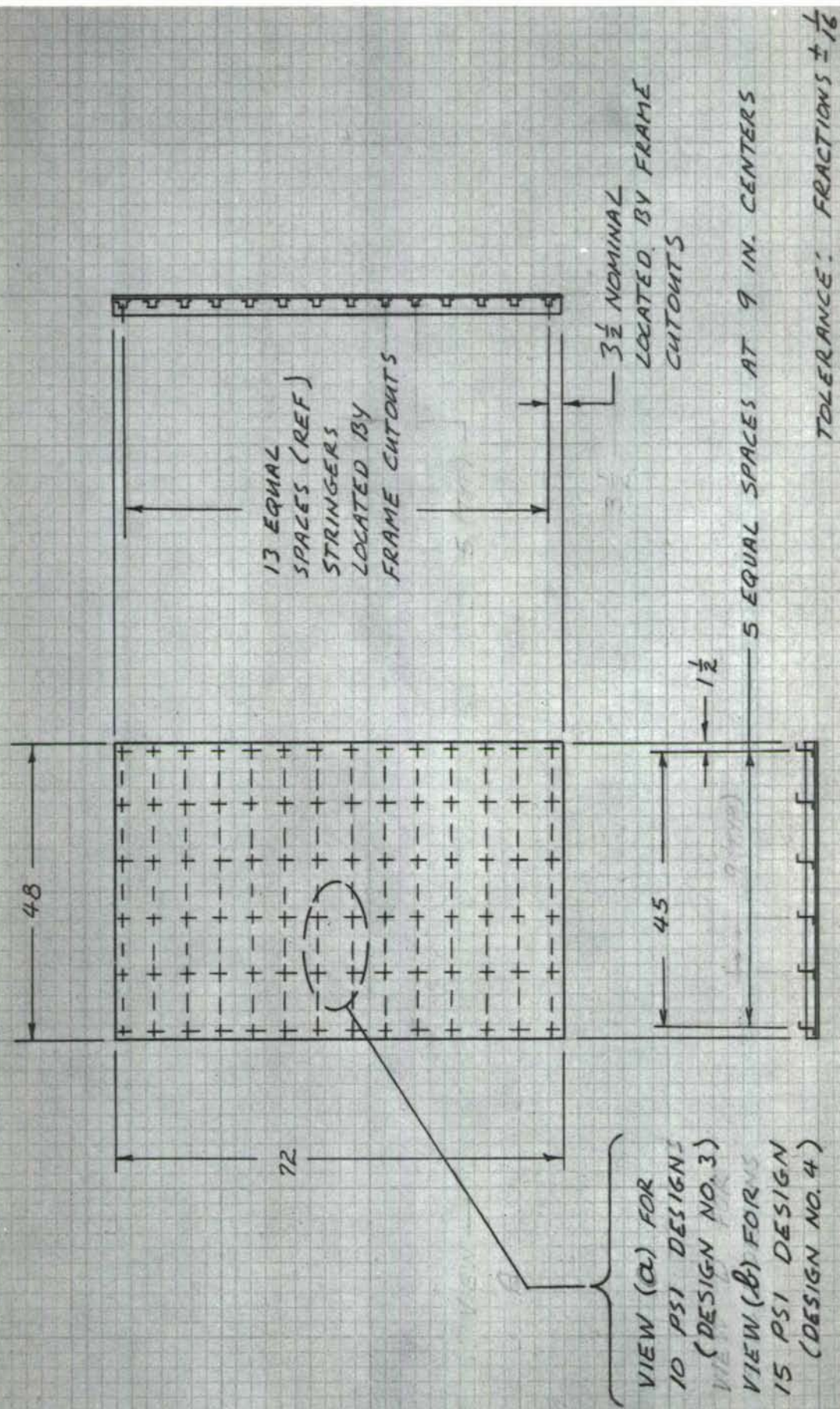
FIGURE 25. FLAT PANELS — HAT SECTIONS, DESIGNS 3 AND 4

13 EQUAL
SPACES (REF)
STRINGERS
LOCATED BY
FRAME CUTOUTS

3½ NOMINAL
LOCATED BY FRAME
CUTOUTS

5 EQUAL SPACES AT 9 IN. CENTERS

TOLERANCE: FRACTIONS ±¼₆

48

72

45

1½

VIEW (a) FOR
10 PSI DESIGN
(DESIGN NO. 3)

VIEW (b) FOR
15 PSI DESIGN
(DESIGN NO. 4)

62
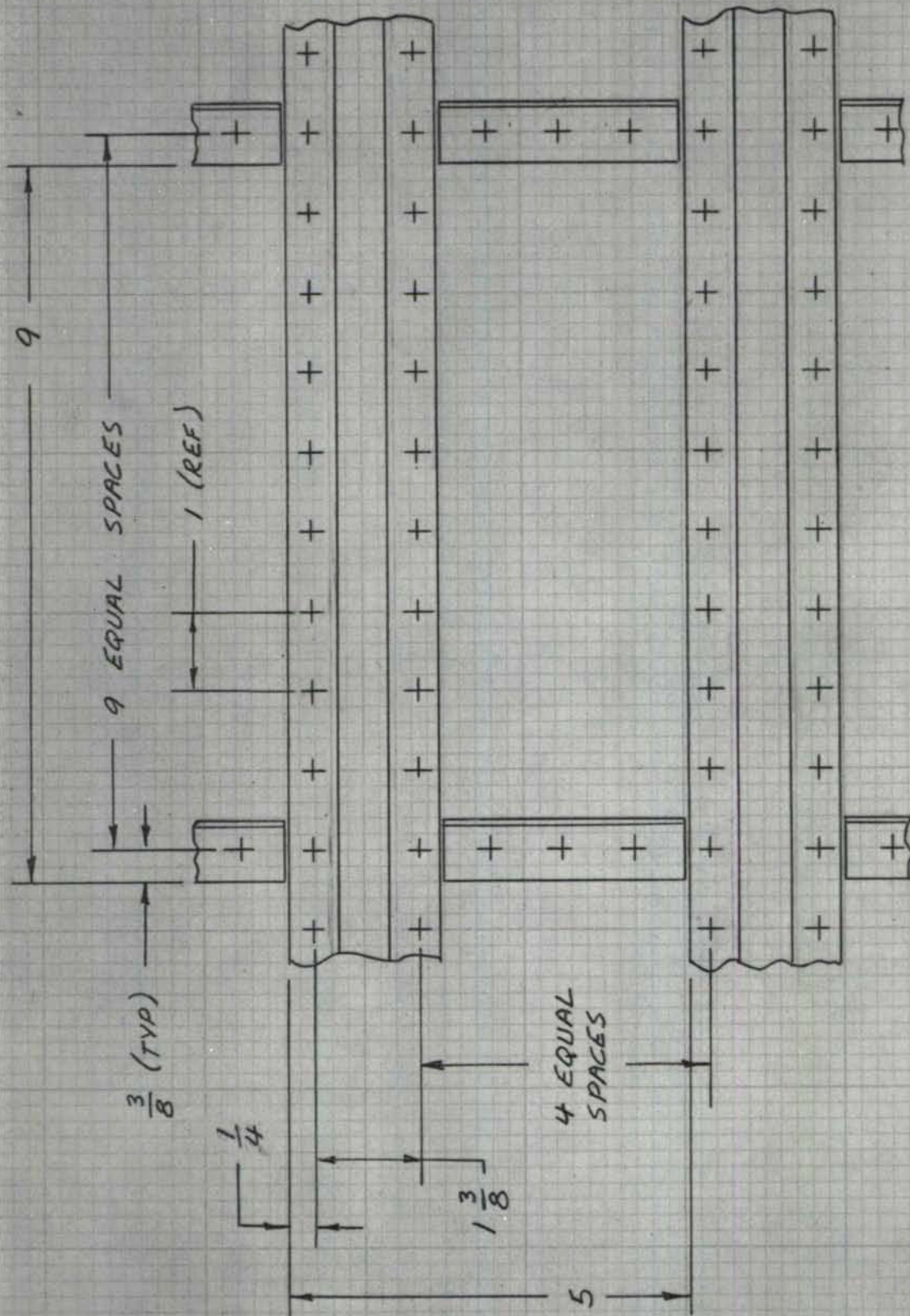
FIGURE 2.5 - CONTINUED
a) 10 P.S.I. DESIGN

63

FIGURE 25 — CONCLUDED
(b) 15 P.S.I. DESIGN

64

13 EQUAL SPACES (REF) STRINGERS LOCATED BY FRAME CUTOUTS

72

60 R

3½

5 EQUAL SPACES IN 45 IN. LENGTH AT 9 IN. CENTERS

1½

48

VIEW (a) SEE FIGURE 24a

TOLERANCE: FRACTIONS ±1/16
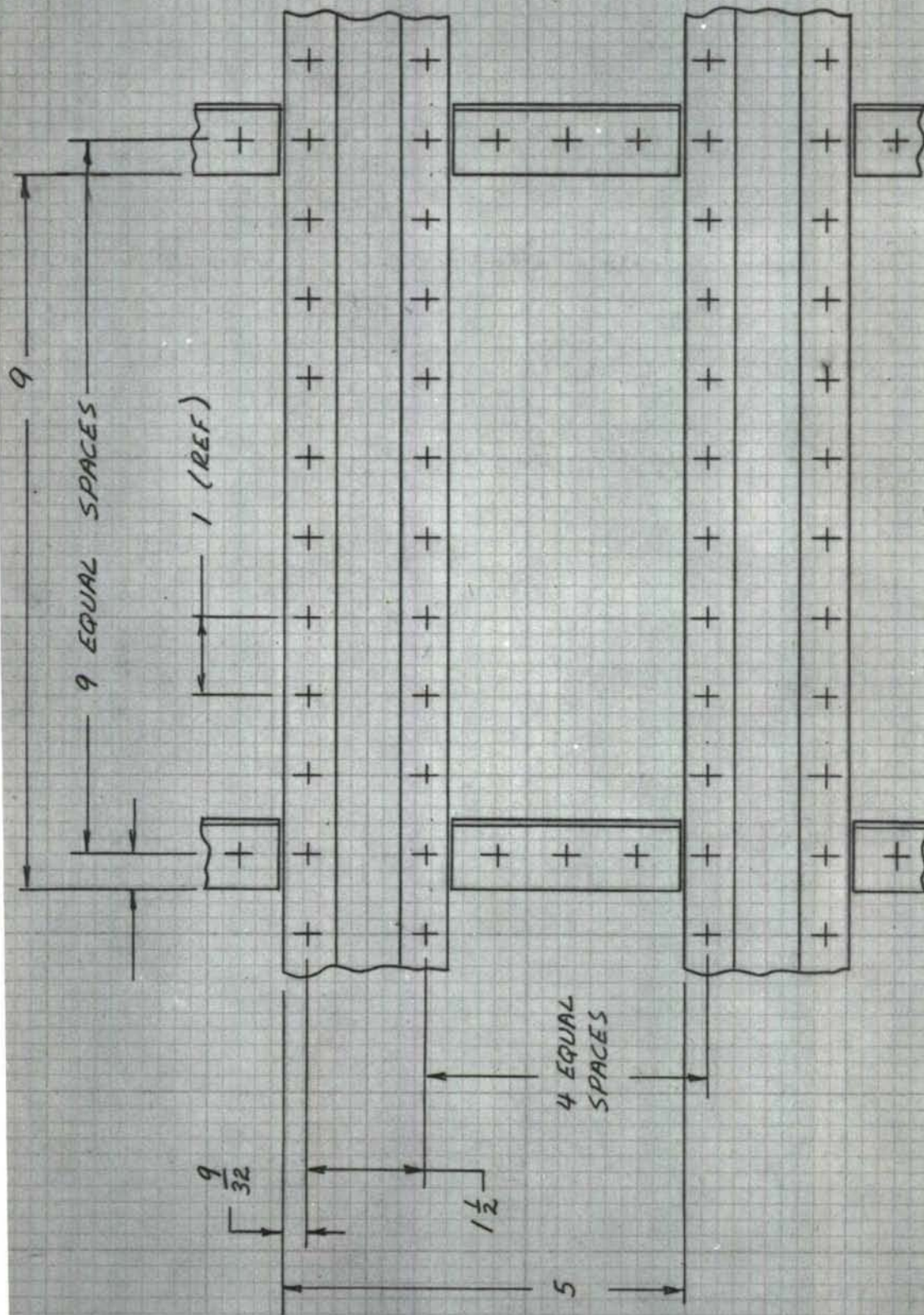
FIGURE 26. CURVED PANELS — J STRINGERS, DESIGN 5

FIGURE 27. CURVED PANEL – HAT STRINGERS, DESIGN 6.

66

4. FABRICATION OF SPECIMENS

4.1 CANTILEVERED BEAMS

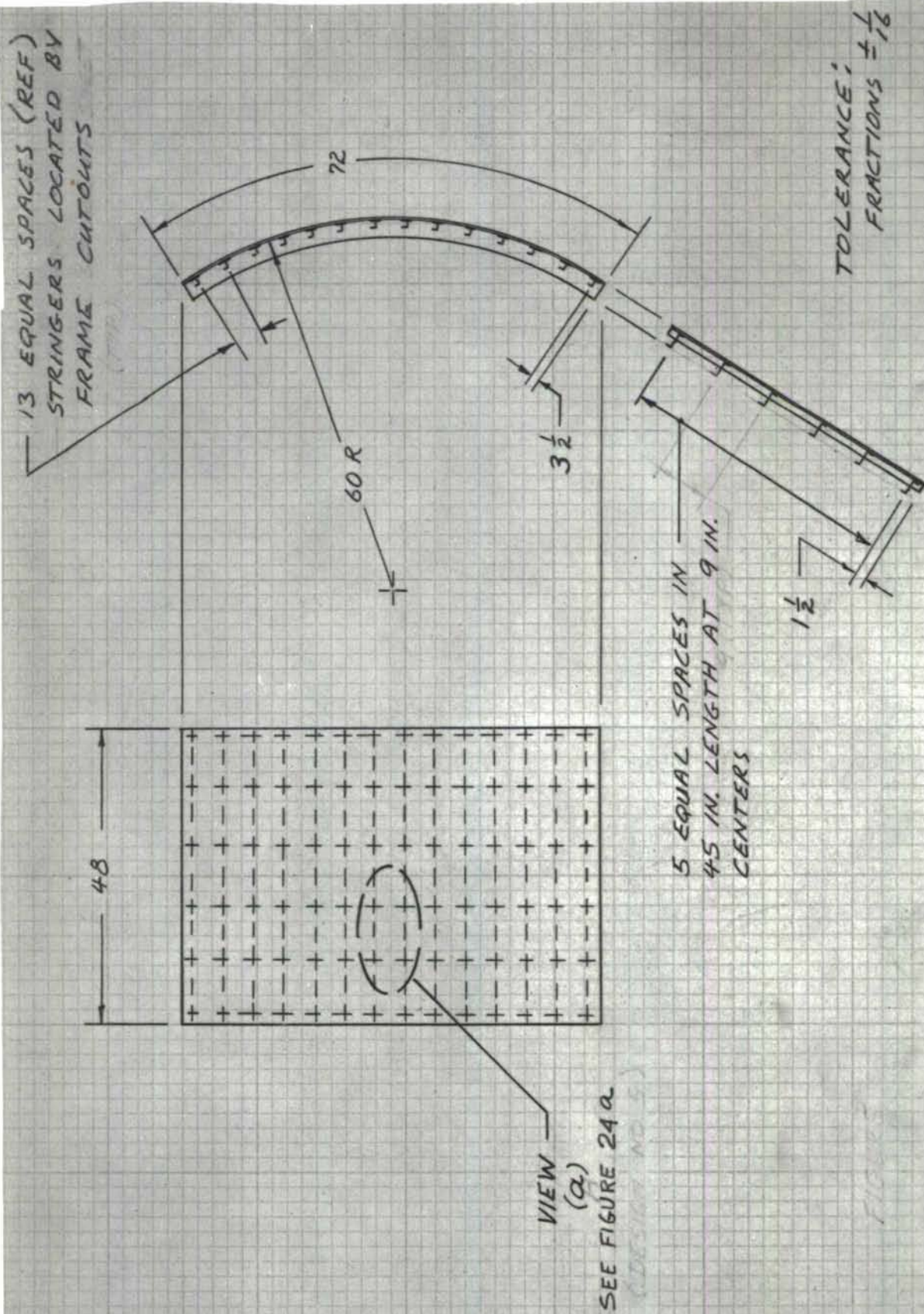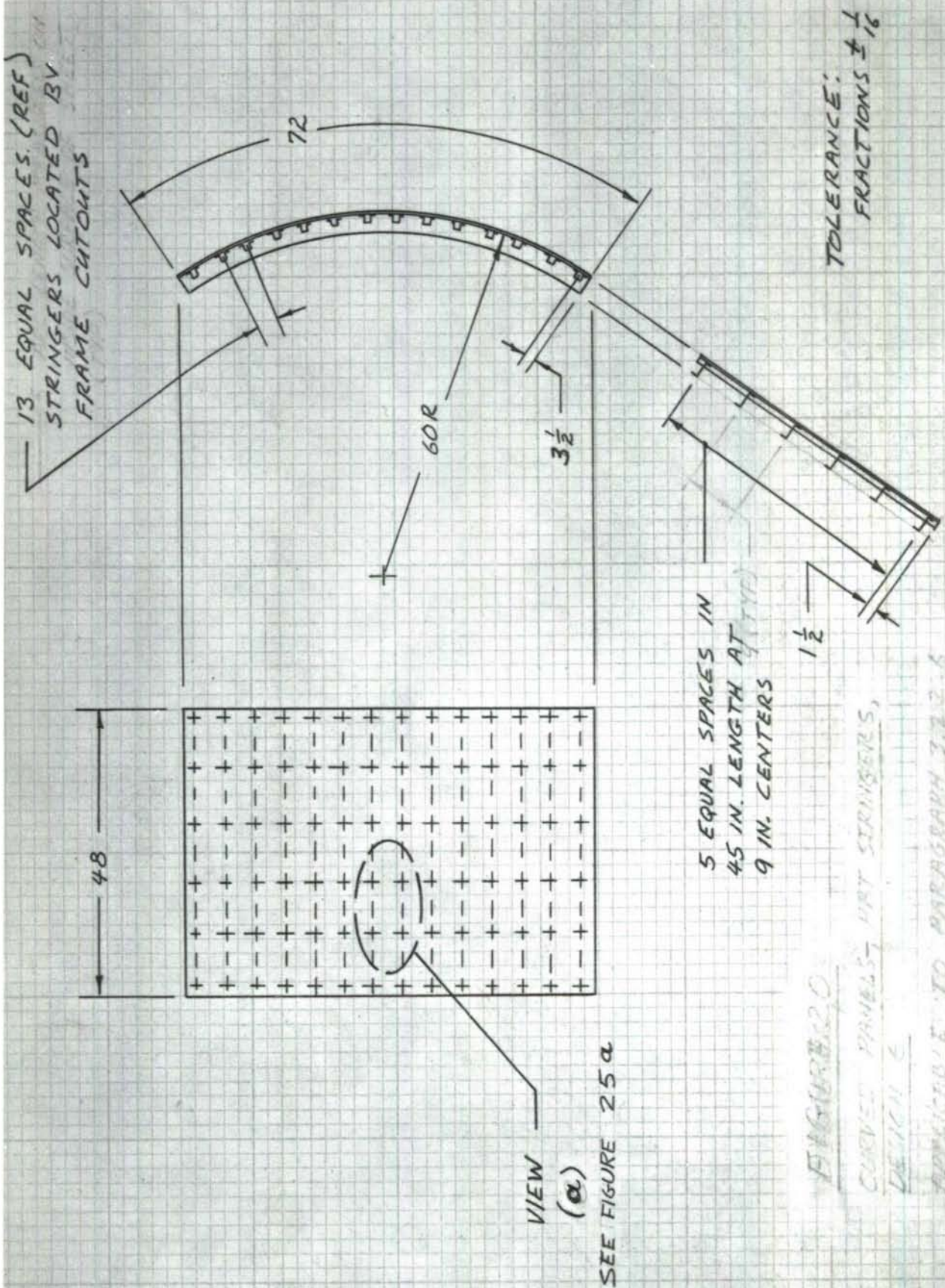The cantilevered beams are made of aluminum sheet as shown in Figure 28. The beam was cut to final dimensions by precision shearing, and the two holes (to be tapped) were stamped out.

The shims were sheared to final dimensions and two holes stamped out with the same die used for the central section. The long surface taper was milled with a side cutting end mill, in a fixture providing rigid backing for the thin material.

The parts were prepared for bonding according to MIL-A-9067C. Assemblies of a beam, two shims and FM-1000 bonding film were then mounted on a fixture with dowel pins, using the holes in the parts for location. Each set of beams going through this cycle was accompanied by two bond shear strength test coupons, of which one was tested to guarantee compliance with the bonding specification, and the other stored.

After bonding and cooling, the extruded beads of bonding agent were milled to leave a skin less than .001 inches thick on the beam.

Completed assemblies were inspected for final dimensions, bonding voids, surface defects and thread tolerance. Cantilevered beams passing inspection were then individually wrapped, and packed in sets.

4.2 TWO BAY PANELS

The two bay panel is shown in Figure 29. All components of the panel were sheared to final dimensions, the rivet holes were stamped.

FIGURE 28. CANTILEVERED BEAM

FIGURE 29. TWO-BAY PANEL

-1 THRU -6 TO BE BONDED TO PLATES PER MIL-A-9067 C. USE .010 THICK FM 1000 BOND

TOLERANCES; DECIMAL: ±.015 IN
FRACTIONAL: ± 1/32 IN

MS 20246AD5 RIVET C'SINK (LG AS REQD) 21 PLCS

69
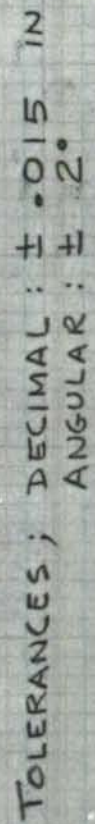
After preparation for bonding per MIL-A-9067C, all surfaces near bonded edges were covered with a self-adhering Mylar tape, and masks of teflon were inserted in the frame. These masks served as pressure pads, but had pressure relief grooves to allow extruded excess bond to flow away from the metal surfaces. A complete set of flat metal parts, bonding film for one panel, and two shear test coupons, were installed in a jig; the whole assembly was processed with a heated platen hydraulic press. Panel assemblies passing bond integrity inspection were completed by riveting on the zee sections. Final inspection and packing for shipping completed this phase.

4.3 THREE BAY PANELS

The three bay panel is shown in Figure 30.

The panels and the flat blanks for the zee sections were sheared to final dimensions; the rivet holes were stamped. Assembly of the three bay panels was completed by riveting the zee sections to the panels.

4.4 FRAME-STRINGER PANELS

The six types of frame-stringer panels and their details are shown in Figures 13. through 27.

The skins were sheared to size and the rivet holes (for the stringers only) were stamped. Clips, with four rivet holes, were stamped and folded into right and left parts. Curved frames were stretch formed. All frames and stringers were cut to size, and rivet holes drilled.

In a typical assembly sequence, the clips were riveted to the stringers, which were then riveted to the skins; the frames, clamped to the skin, served as drilling templates for the skin. After removal of clamps, riveting completed the assembly.

FIGURE 30. THREE BAY PANEL
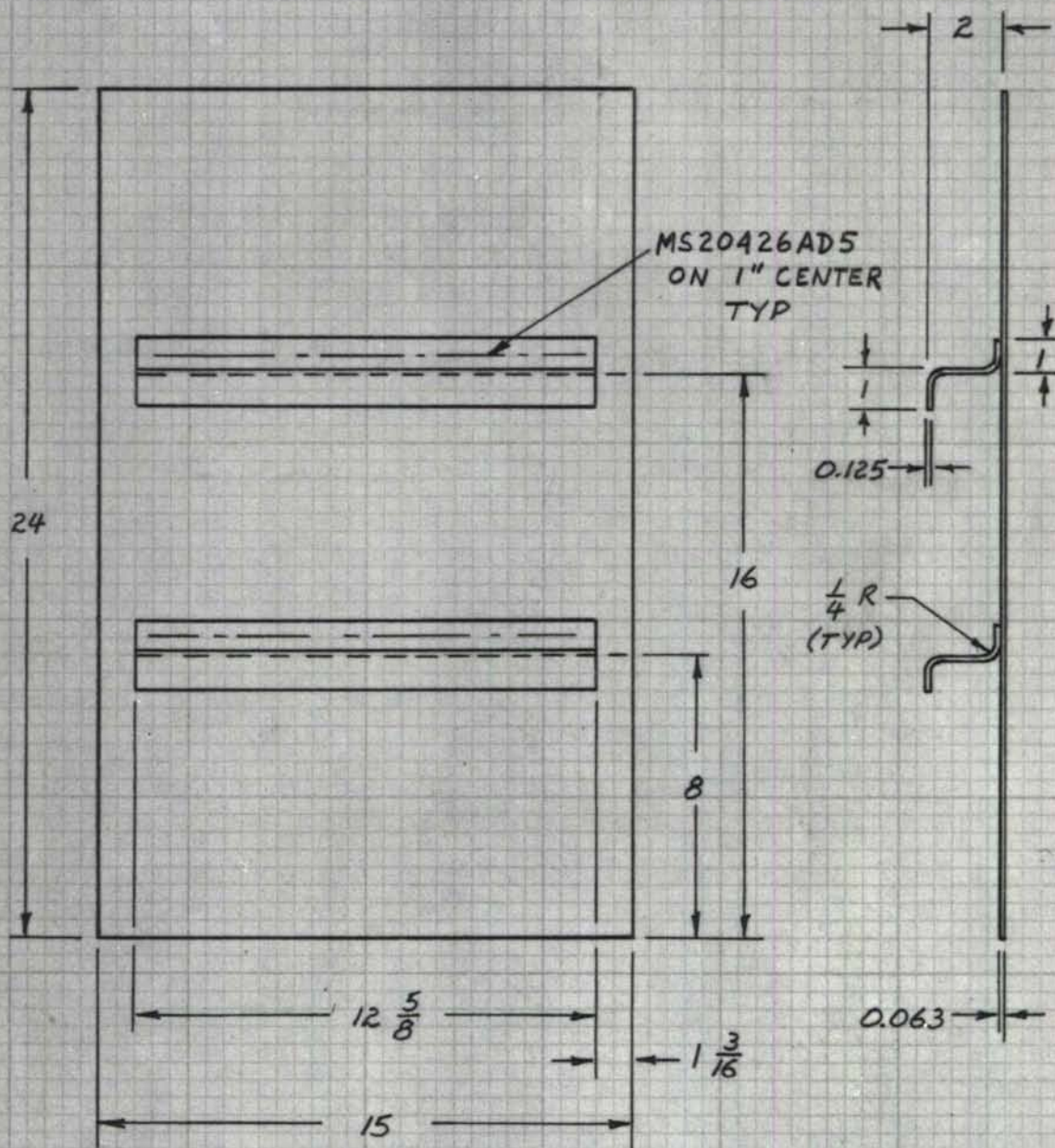
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13. ABSTRACT

This report describes the analysis, design and fabrication of structural specimens to be used in an investigation of peak pressure and multi-mode effects of sonic fatigue. These specimens are to be fatigue tested to failure in the RTD Sonic Fatigue Facility by RTD personnel.

Details of the analysis and design are presented for three-bay panels and frame stringer panels. The three-bay panels were designed to fatigue in 10^5 cycles when exposed to an SPL of 162 db. A linear structural response approach is used, and the possibility of nonlinearities is examined.

The frame stringer panels were designed to withstand various combinations of internal bursting pressures and external aerodynamic loads.

Procedures used in fabricating the above specimens, and specimens designed by R&D personnel are also described.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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